

On the Definition and Measurement of Ageing*

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Abstract

Ageing is central to economic and social policy, yet there is no widely accepted, operational definition of ageing. This paper proposes a broad, intuitive, and testable definition based on two axiomatic conditions: monotonicity and inevitability. These require that ageing variables deteriorate in expectation over long horizons, while allowing for short-run reversals, and that deterioration becomes increasingly certain as the time horizon expands. We formulate these conditions at the individual, cohort, and population levels and derive hypothesis tests to assess whether observed variables satisfy them. We apply this framework to panel data from the Study of Health, Ageing and Retirement in Europe (SHARE) and English Longitudinal Study of Ageing (ELSA), using dynamic panel models that allow for persistence and potential non-stationarity. Examining a range of health and economic outcomes, we find that variables which appear to reflect ageing at the population level may not do so at the individual level. Moreover, most heterogeneity in later-life outcomes is unrelated to ageing and instead reflects differences established earlier in life. Finally, we show that even when variables satisfy the proposed ageing conditions, they are generally not invertible, implying that measures of “biological age” can be highly misleading.

Keywords: Ageing, Longevity, Health, Inequality

JEL Codes: C12, C23, I14, J14

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1 Introduction

As life expectancy increases and the percentage of the population in older years rises, there is a growing interest in a broad range of issues linked to ageing across multiple disciplines (biology, health, economics, finance, sociology). The sheer breadth of this research points to the notion that ageing is a common phenomenon reflected in a wide range of variables. In spite of this, there is no generally accepted definition of ageing (Balcombe and Sinclair, 2001; Cohen, Legault and Fülöp, 2020). Scientific and social developments are changing the relationship between chronological age and almost any observable medical or economic indicator, and the macroeconomic and individual consequences of this changing relationship are wide-ranging and substantial. Developing an empirically and theoretically workable definition of ageing is therefore of first-order relevance for economics, as it is necessary to conceptualise the economic costs and opportunities presented by ageing at both the individual and social levels. For example, being able to distinguish between poor health and ageing is vital for health and labour market policies. Understanding and being able to separate ageing from other factors is also crucial for measuring and addressing inequality and heterogeneity across individuals.

In this paper, we introduce a general axiomatic definition of ageing that can inform academic and policy discussions. Our definition is based on two conditions: monotonicity and inevitability. At the individual level, these conditions require that ageing variables exhibit long-run deterioration in expectation (Individual Monotonicity), while allowing for short-term improvements and reversals, and that deterioration becomes increasingly certain as the time horizon expands (Individual Inevitability). In other words, ageing progresses in a consistent direction over time and it eventually happens to everyone. We formulate parallel conditions at the cohort level and for commonly used cross-sectional population measures, highlighting the potential disconnects between individual ageing processes and population-level patterns. Based on these conditions, we propose hypothesis tests for whether individual observed variables satisfy these conditions and should be thought of as an “ageing variable.”

We then bring this framework to data using panel data on health and economic outcomes from the Study of Health, Ageing and Retirement in Europe (SHARE) and the English Longitudinal Study of Ageing (ELSA). We examine whether widely used candidate measures (e.g. frailty, functional disability, pain, mental health, and income) satisfy the proposed ageing conditions. We use dynamic panel models that account for potential non-stationarity in order to characterise individual and population trends and potential reversibility in these variables. This allows us to systematically apply our definition and empirically test whether different measures qualify as ageing variables. We then explore the implications of our definition for a range of practical issues that concern economists. This includes the distinction between population, cohort and individual measures of ageing; identifying the contribution of ageing to explaining the heterogeneity in outcomes among older adults; and the unreliability of measures of biological age.

Our findings include four key results. First, we demonstrate how the distinction between individual, cohort and population conceptions of ageing is crucial. Some variables may appear to be driven by ageing when assessed at a cross-sectional population level, but not at an individual or cohort level. Second, we find substantial heterogeneity in health and economic outcomes unrelated to age. In many cases, we find that the vast majority of variance in health and economic variables across older adults is not explained by ageing, but rather their level in middle age. This suggests that much of the economic focus on ageing is misplaced and should rather focus on inequality earlier in life. Third, the high proportion of age-independent heterogeneity means that while the deterioration of ageing variables may be inevitable, this is only the case over very long horizons, which has encouraging implications for the reversibility and malleability of ageing processes. Finally, we also find that ageing variables are generally not reliably invertible. In other words, extracting an implied (or “biological”) age, even from variables that satisfy our ageing conditions, can lead to very misleading results.

While a variety of measures of ageing have been proposed, there has been little discussion of the necessary conditions for something to constitute an “ageing” variable. The key idea underlying many biological definitions of ageing is a process of physiological decline that leads to higher vulnerability and mortality as it progresses. This is often framed in terms of biological features such as telomere attrition, genomic instability or a loss of

proteostasis. For example, [López-Otín et al. \(2023\)](#) identify twelve “hallmarks” of ageing but these are better seen as characterising mechanisms or indicators of ageing, rather than defining ageing itself. [Cohen, Legault and Fülöp \(2020\)](#) argue that there is no unitary biological phenomenon that corresponds to the word “ageing”, but rather “an assortment of loosely related processes that we find convenient to lump together.” Similarly, [Golubev \(2021\)](#) draws a distinction between nominal and real definitions of ageing, where the nominal definition of ageing consists of a set of observable features while a real definition (tied to underlying mechanisms) is lacking. While [Cohen, Legault and Fülöp \(2020\)](#) advocate a gradual abandonment of the term, we instead aim to provide a definition that is flexible enough to potentially cover this assortment of loosely related processes, while still having sufficient content to be testable and falsifiable.

While biological definitions of ageing generally aim to either explain the underlying mechanisms of ageing, they do not try to define what ageing itself is. This is more of a philosophical or conceptual question, and it is in this tradition that this paper principally sits. In a wide-ranging review, [Lemoine \(2020\)](#) identifies five features that are commonly used to define ageing: structural damage, functional decline, depletion, typical phenotypic changes or their underlying causes, and increasing probability of death. Similarly, [Horvath and Raj \(2018\)](#) identify that valid ageing biomarkers must satisfy several implicit conditions: increasing monotonically within individuals, predicting mortality, and be based on underlying biological processes. These are generally plausible features, but it is not clear that they are either necessary or sufficient to characterise ageing in an economic context.¹ Another philosophical approach asks whether aging should be classified as a disease ([Saborido and García-Barranquero, 2022](#)), but this leaves unanswered the question of what the properties of ageing are that distinguish it from other diseases.

A common factor across many conceptions or definitions of ageing is an ability to predict mortality. While this does have attractive properties, not least that mortality is perhaps the most easily and reliably measured biomarker, it is not desirable to tie our definition of ageing to mortality for economic questions. Firstly, there may be variables or properties that we would want to include as ageing separately from their mortality effects. In the social sciences, ageing is often defined as a multidimensional concept that includes social, psychological and cultural aspects ([Menassa et al., 2023](#)). Sociological work has argued that ageing is partly defined through social reality rather than just biology ([Gilleard, 2025](#)), or that ageing is a dynamic interplay between biological changes and social environments ([Wejbrandt, 2014](#)). Secondly, a crucial concern from both a macroeconomic and individual welfare perspective is that the link between ageing and mortality is weakening or breaking. The widely-debated economic challenges posed by an ageing society often presuppose that a large portion of society will live much longer in an aged state.

Our axiomatic definition of ageing is consistent with the term’s colloquial use, while remaining agnostic on (the existence of) an underlying mechanism. Our two conditions, or monotonicity and inevitability, imply an understanding of ageing as a property that certain measurable variables may or may not have. For example, a measure of frailty might have the property of being an “ageing” variable, while a measure of subjective wellbeing might not. Our definition is thus consistent with social constructivist accounts of ageing as well as any based on fundamental biology. In this sense, our work is closest to those that propose mathematical conceptions of ageing such as a loss of complexity ([Weon and Je, 2010](#)), increased system failure risk and redundancy loss ([Gavrilov and Gavrilov, 1991](#)), or linked to an increasing mortality rate over time ([Qin, 2024](#)).

The remainder of this paper is structured as follows. Section 2 states our conditions mathematically and discusses the distinction between individual, cohort and population average versions of these conditions. Section 3 introduces our data and presents some initial illustrative evidence. Section 4 introduces our empirical strategy: deriving test statistics for the conditions introduced in Section 2 and discussing issues such as non-stationarity. Section 5 applies our tests to several example variables and discusses the results. Section 6 then discusses some important implications of these ageing measurements and tests for economic analysis.

¹A related literature explores the evolutionary origins of ageing, often based on the idea that ageing results from a decline in the intensity of natural selection with chronological age ([McAuley, 2025](#)).

2 Theory

We propose two conditions to capture intuitive properties of ageing: that ageing occurs over time, and that at some point we all must age. These conditions allow for temporary reversals of ageing, for example as the result of lifestyle changes, but do not allow for permanent improvements in ageing. That is to say that ageing can be delayed but not avoided.

2.1 Ageing variable conditions

2.1.1 Individual conditions

The first condition we proposed is Individual Monotonicity (IM). In expectation, (normalised) ageing variables for an individual will increase in value over time, conditional on survival. Formally:

$$\exists M \text{ s.t. } \tilde{E}(y_{i,a+j} - y_{i,a} \mid y_{i,a} < y^{max}) > 0, \forall j > 0, a \geq M \quad (\text{IM})$$

That is, there exists some age of maturity (M) past which the ageing variable, in expectation and conditional on survival, increases at every age past M . This condition allows for fluctuations in the values of the ageing variable (for example, as the result of lifestyle change) but requires that, on average, it increase over time. Here \tilde{E} denotes the expectation conditional on individual i surviving to age $a + j$. We also condition on the ageing variable not beginning in at some sufficiently extreme state; for example, if the variable under consideration is the ability to walk and an individual cannot stand at a , then they would not meet the initial condition.

In principle, j can take any positive value; in this paper, unless otherwise noted, we will use it to represent 1 year. Many surveys are conducted on an annual basis, making 1 year a practical time difference; this also allows us to disregard any seasonal effects.

The second condition we propose is Individual Inevitability (II). As the future time horizon increases, the impact of ageing becomes inevitable and holds not just in expectation but with certainty.

$$\exists H > 0 \text{ s.t. } \tilde{P}r(y_{i,a+j} > y_{i,a} \mid y_{i,a} < y^{max}) \rightarrow 1 \text{ as } j \rightarrow H, \text{ for } a \geq M \quad (\text{II})$$

As an individual ages, the probability that an ageing variable is worse than at a previous age approaches 1. This ageing takes place over the time horizon H . While IM allows for short-term improvements in ageing variables, II reflects the notion that ageing is unavoidable in the long-run. Note that a smaller value of H implies a stricter condition (i.e. a shorter timeframe over which the variable must get deteriorate).

2.1.2 Cohort conditions

Following on from the individual conditions, we consider how ageing variables must evolve over time for a given cohort. Following immediately from the individual results, conditioning on survival to time $t + j$, we arrive at the below cohort conditions:

$$\tilde{E}(\bar{y}_{t+j}(a+j) - \bar{y}_t(a)) > 0, \forall j > 0, a \geq M \quad (\text{CM})$$

Cohort monotonicity states that the expected average value of an ageing variable, $\bar{y}_{t+j}(a+j)$ for those age $a + j$ (and still alive) at $t + j$, must be at least as large as the average value for those same individuals when they were age a at time t . Here, \tilde{E} restricts the individuals included in $\bar{y}_t(a)$ to those still alive at $t + j$.

$$\tilde{P}r(\bar{y}_{t+j}(a+j) > \bar{y}_t(a)) \rightarrow 1 \text{ as } j \rightarrow H, \text{ for } a \geq M \quad (\text{CI})$$

In expectation, an ageing variable for a cohort must increase over time. As the cohort approaches the maximum lifetime, this happens with certainty. However, conditioning on survival to $t + j$ reduces the practical value of these measures: at time t , it is not known which individuals will survive to $t + j$.

2.2 Condition extensions

2.2.1 Average measures

It is also common in both the policy and academic literature to compare health or ageing status cross-sectionally². However, ageing variables satisfying the individual conditions need not have the analogous properties in a cross-sectional sample.

The analogous cross-sectional monotonicity condition states that, at any time, an ageing variable must take a higher value for those at higher ages. In addition to the mortality selection described above, averages between cohorts may differ if the underlying ageing process changes over time.

$$E(\bar{y}_t(a+j) - \bar{y}_t(a)) > 0, \forall j > 0, a \geq M \quad (\text{AM})$$

Similarly, the analogous average inevitability condition is as follows:

$$\Pr(\bar{y}_t(a+j) > \bar{y}_t(a)) \rightarrow 1 \text{ as } j \rightarrow H, \text{ for } a \geq M \quad (\text{AI})$$

As the age difference increases, the probability that the older average must be higher goes to 1.

Without making additional assumptions on the change in the ageing process over time and on mortality selection, the average conditions do not follow from the previous individual and cohort conditions³.

2.3 Discussion

The horizon H , maturity age M , and therefore what constitutes an ageing variable can change over time. For example, medical advances may slow or stop the progression of some (current) measures of ageing. Thus, these are tests for ageing at a particular moment in time and space. Measures that were previously ageing variables may cease to be so with medical advances that provide treatment.⁴ The dynamic nature of ageing across time and space underscores the importance of a rigorous axiomatic definition of ageing. Furthermore, the conditions we propose are necessary but not sufficient conditions for a variable to be considered for measures of biological age. A variable which does not satisfy these properties cannot be inverted to infer an age equivalent.

3 Data and descriptives

To illustrate these conditions, we use data from two nationally representative longitudinal panel studies that follow the health, economic circumstances, and social well-being of individuals aged 50 and older: the English Longitudinal Study of Ageing (ELSA) and the Survey of Health, Ageing and Retirement in Europe (SHARE). Both studies collect biennial data from the same individuals, providing extensive longitudinal records with low attrition rates, ensuring sufficient statistical power to detect meaningful changes in health trajectories. The empirical analysis uses the harmonised versions of ELSA (waves 1–10, 2002–2019) and SHARE (waves 1–9, 2004–2023), both of which provide longitudinally consistent coding of health, functioning, and demographic measures within a uniform structure. The unit of observation in each dataset is the individual–wave, identified by a unique respondent identifier and survey wave. Across both studies, the analysis draws on comprehensive assessments spanning multiple domains: physical health (including objective performance measures such as grip strength and walking speed), functional and mobility limitations, pain, cognitive functioning, mental health, and detailed socioeconomic information, as well as anthropometric and biomarker indicators. These measures combine self-reported data, nurse-conducted health assessments, and, in the case of ELSA, administrative

²Examples include [Case and Deaton \(2005\)](#), [Rockwood and Mitnitski \(2007\)](#), [Banks et al. \(2006\)](#).

³Consider a new environmental toxin that adversely impacts a variable that meets the individual conditions, but only for those exposed in childhood. Then, the first cohorts who are exposed in childhood could have worse outcomes than older cohorts, despite their younger age.

⁴As an example, unaided visual acuity would likely have been an ageing variable in the past, but the advent of LASIK in the 20th century made permanent improvements above baseline possible.

linkages, though collection frequency varies across waves and measurement protocols. Together, ELSA and SHARE provide a rich comparative foundation for dynamic panel data analysis, enabling investigation of how health outcomes evolve within individuals over time across distinct national and institutional contexts. This breadth makes them particularly well-suited for studying complex patterns of health deterioration and recovery among ageing populations and for constructing outcomes that meet the conceptual and empirical criteria for ageing variables set out in this study.

3.1 Common ageing variables

Table 3.1 presents descriptive statistics for the key health and economic variables used in our analysis of ageing trajectories in the SHARE and ELSA datasets. The health measures include mental health symptoms, functional disability, a composite frailty index, and a pain index. We have normalised or obtained all health measures to the range $[0, 1]$, where higher values indicate poorer health status. The economic variable captures individuals’ financial well-being through real gross income (including all income sources), measured in either Euros or British pounds and adjusted for inflation. These variables collectively capture the multidimensional nature of ageing, encompassing physical health, mental well-being, functional capacity, and economic security across the life course. Lastly, to avoid extreme values recorded for real gross income, we winsorise the data for this variable to the 1st and 99th percentiles.

Table 3.1: Descriptive Statistics: Health and Economic Variables

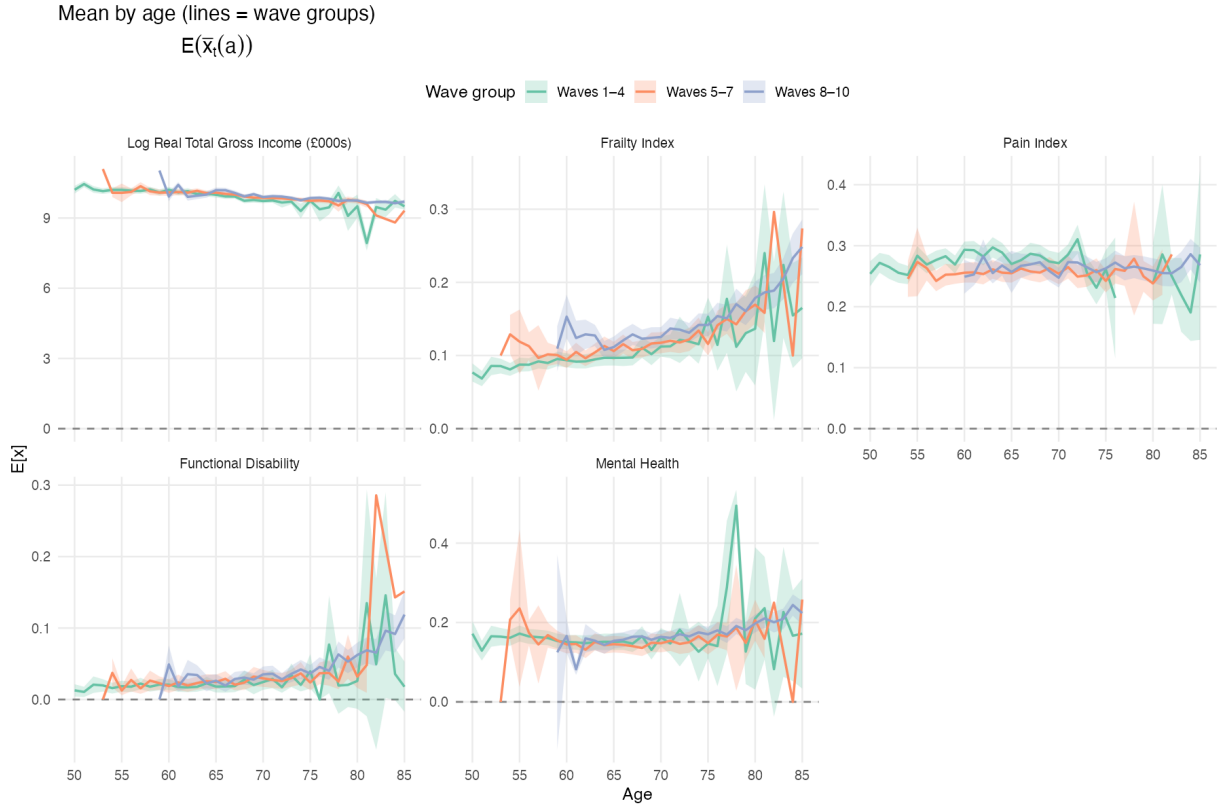
Variable	Obs	Mean	Std. Dev.	Min	Max
Real Gross Income (£,€)	167,846	45,660	74559.51	200	400000
Frailty Index	442,171	0.13	0.15	0.00	1.00
Pain Index	130,473	0.26	0.15	0.14	1.00
Functional Disability	436,149	0.05	0.16	0.00	1.00
Mental Health Symptoms	425,228	0.19	0.21	0.00	1.00

3.2 Additional descriptives and considerations

3.2.1 Cross-sectional results

To provide intuition for the conditions and show how they appear in the data, we now present illustrative charts. First, we consider how the mean evolves across different ages and over time for the candidate variables we consider. Each panel of Figure 1 shows the mean value of the variable, with 95% confidence bands, by age and wave. The shape of the lines over time can be interpreted as the cross-sectional variation in each variable, i.e. the “Average” measure described in Section 2.2.1. The health variables are all relatively stable across waves, while log real total gross income is modestly higher in later waves. Both Frailty and Functional Disability are increasing in age in the cross-section, whereas the Pain Index is relatively flat and Mental Health is non-monotonic across the age distribution.

Figure 1: Confidence interval for the mean of each variable, by age



3.2.2 Individual monotonicity

Now, we consider how the values of each variable change *for each individual* over the life course. Each panel of Figure 2 shows the mean four-year (two-wave) change in the value of the variable across individuals (i.e. $j = 4$ in equation IM). The darker shaded region shows the 95% confidence bands for the mean, while the lighter bands show the 10th to 90th percentile range. Age is on the horizontal axis, and each panel shows how the variable changes for individuals starting at different ages (i.e. different values of t in equation IM). The means for both the Frailty Index and Functional Disability are statistically different from 0 for all ages above 65, and Functional Disability is (weakly) monotonic even in the 10th percentile. As was seen in the cross-section, the Pain Index is not statistically different from 0 and Mental Health is non-monotonic. The mean change in log real total gross income is close to 0 across the full age range, with wide confidence bands reflecting substantial individual-level heterogeneity in income trajectories.

3.2.3 Individual inevitability

Next, we turn to an illustration of the inevitability for individuals. Figure 3 shows the share of individuals (excluding those already at the extreme value in the base year) who are strictly worse off $2 * j$ years later. The x-axis is two years periods since the initial measurement and the y-axis is the worse off share. The coloured bands show different age groups.

For income, we see that for those initially in their 50s, the probability of decreasing real gross income increases over time, corresponding to a reduction in income when stopping work. For those in their 60s and 70s, the probability is relatively flat, which could be indicative of an initial drop and then flattening, or of fluctuations in income unrelated to time. For the frailty index, all cohorts see that the probability of increased frailty increases with time, with higher levels at older ages. For all cohorts, the probability of being worse off in 16 years than they were initially is above 80%. In contrast, the probability of an increase in the pain index is much lower, and the estimates are noisier. While the probability of increasing pain increases as the horizon

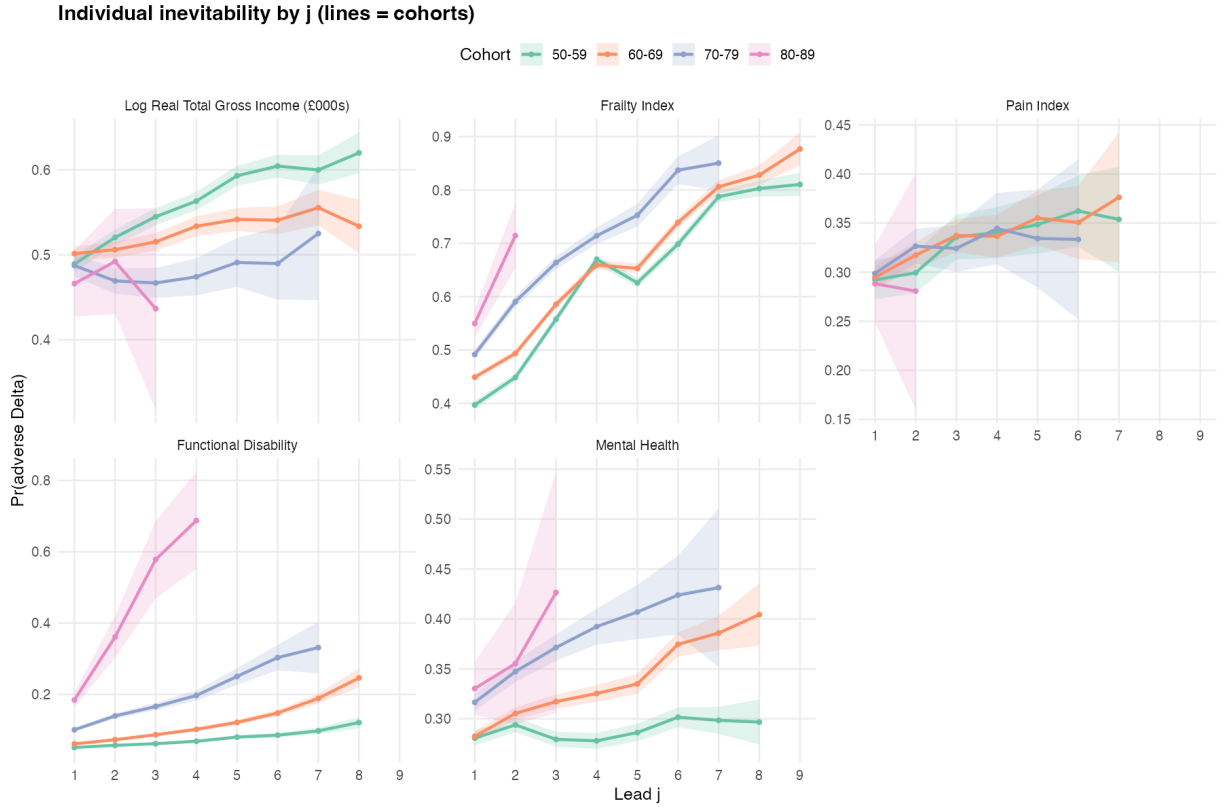
Figure 2: Confidence interval for the mean four-year change, by age



lengthens, it is similar across cohorts. Relating this to the condition, this means that increasing M may not reduce the horizon H over which deterioration is inevitable.

For both functional disability and mental health, we see relatively low and flat probabilities of deterioration for the younger cohorts, but a steep increase for those in the older cohort. In the last periods, more than 50% of individuals are strictly worse off than they were initially. This implies the age of maturity may be somewhat later for these variables, but the trajectory does point toward eventual inevitability.

Figure 3: Confidence interval for the share with a strict increase (decrease) in level of ageing (income) variables



4 Empirical Strategy

To test our conditions, we consider the following model:

$$Y_{i,t} = \phi Y_{i,t-1} + \alpha_i + \epsilon_{i,t} \quad (1)$$

This model - if estimated in levels - estimates the persistence of the ageing-variable candidates from our dataset. However, it is important to note that the regressions in levels may demonstrate a unit root, which we also formally test in the following discussion.

4.1 Stationarity

To assess the time series properties of each outcome variable, we conduct panel unit root tests following the Im-Pesaran-Shin (IPS) methodology (Im, Pesaran and Shin, 2003). For each individual i , we estimate an Augmented Dickey-Fuller (ADF) test with one lag, testing $H_0 : \rho_i = 1$ against $H_1 : \rho_i < 1$, and aggregate the individual p -values using Fisher's combined probability test:

$$\chi^2 = -2 \sum_{i=1}^N \log(p_i) \quad (2)$$

which follows a chi-squared distribution with $2N$ degrees of freedom under the null.

The panel-level Fisher statistic, however, can be misleading in the presence of individual heterogeneity. Because the statistic aggregates $\log p$ -values, a minority of individuals with very small ADF p -values can dominate the aggregate and produce a large test statistic even when the majority of individuals are non-stationary. For all variables, the Fischer test rejects the null hypothesis of a unit root. Table 4.1 illustrates that

despite this result for all variables, between 68% and 83% of individuals retain a unit root depending on the variable⁵.

The individual-level results therefore provide the more relevant picture. For most individuals, we find a unit root in levels but stationarity in first differences, indicating that the series are most likely integrated of order one ($I(1)$). This justifies our first-difference specification and confirms that it avoids spurious regression problems. Furthermore, the first-differences approach remains appropriate even for stationary individuals.

Table 4.1: Unit Root Test Results by Ageing Variable

Ageing Variable	% with UR	% Stationary
Log Real Total Grs Inc	67.6	32.4
Frailty Index	71.2	28.8
Pain Index	72.7	27.3
Func Disab	82.6	17.4
Mental Score	75.9	24.1

Following these results, we estimate the dynamic panel specification in first differences:

$$\Delta Y_{i,t} = \beta \Delta Y_{i,t-1} + \alpha_i + \omega_t + u_{i,t} \quad (3)$$

where $Y_{i,t}$ is an ageing variable, β is a common autoregressive slope, α_i is an individual fixed effect, and ω_t is a wave fixed effect.

As an extension, we stratify individuals by their health trajectory direction, classifying each person as exhibiting an upward trend (deteriorating health), downward trend (improving health), or flat trend based on the significance ($p < 0.10$) of a linear time trend. Separate unit root tests for each trajectory type allow us to examine whether health shocks represent permanent changes (unit root) or temporary deviations from an underlying trend (trend-stationarity), and whether this differs systematically between deteriorating and improving trajectories. In Section 5.2, we will discuss how this exercise can show potential malleability trends for our candidates of ageing variables.

4.2 Testing the Conditions

To bring the theory to data requires some minor adjustments to II and CI so they are testable in a sample.⁶

$$\exists H > 0 \text{ s.t. } \tilde{Pr}(y_{i,a+j} > y_{i,a} | y_{i,a} < y^{max}) \rightarrow 1 - \eta \text{ as } j \rightarrow H, \text{ for } a \geq M$$

$$\tilde{Pr}(\bar{y}_{t+j}(a+j) > \bar{y}_t(a)) \rightarrow 1 - \eta \text{ as } j \rightarrow H, \text{ for } a \geq M$$

Rewriting these in terms of differences, we get:

$$\exists H > 0 \text{ s.t. } \tilde{Pr}(\Delta^j y_{i,t+j} > 0 | y_{i,t} < y^{max}) \rightarrow 1 - \eta \text{ as } j \rightarrow H, \text{ for } t \geq M \quad (4)$$

$$\tilde{Pr}(\Delta^j \bar{y}_{t+j}(a+j) > 0) \rightarrow 1 - \eta \text{ as } j \rightarrow H, \text{ for } a \geq M \quad (5)$$

Here, for Inevitability, we add a threshold, $1 - \eta$, and formulate the condition as the existence of H . Since the ELSA sample starts at age 50, we first consider whether the conditions are satisfied when $M = 50$. Note

⁵The distribution of individual ADF p -values (Appendix Figure 10) confirms this pattern, exhibiting fat tails that drive the aggregate statistic without reflecting the typical individual in the sample.

⁶Note that the conditions are formulated for variables for which an increase in value corresponds to a worse outcome; for the income variable in our analysis, the inequality is reversed.

that while the individual conditions imply those for the cohort, for inevitability, the requisite threshold may be crossed for lower values of H than for individuals. Throughout this section, we take the parameters estimated in the regressions as the true values, and so abstract from estimation uncertainty.

Analogously, we can write the IM in terms of differences.

$$\exists M \text{ s.t. } \tilde{E}(\Delta^j y_{i,t+j} | y_{i,t} < y^{max}) > 0, \quad \forall j > 0, t \geq M \quad (6)$$

For the hypothesis testing, we use the Random Effects models (i.e. $\alpha_i = \alpha \forall i$), as α_i cannot be consistently estimated in a fixed-length panel. Thus, individual differences in ageing are captured in $u_{i,t}$ and the variation enters the testing through σ .

4.2.1 Distribution of cumulative changes

Consider the process

$$\Delta Y_{i,t+1} = \alpha + \beta \Delta Y_{i,t} + u_{i,t}, \quad (7)$$

with $\{u_{i,t}\}$ i.i.d. $N(0, \sigma^2)$ and independent of the initial condition. For a fixed horizon $j \geq 1$, define the cumulative change

$$C_{i,t}(j) := Y_{i,t+j} - Y_{i,t} = \sum_{h=1}^j \Delta Y_{i,t+h}. \quad (8)$$

4.2.2 Conditional distribution

For $\beta \neq 1$, iterating (7) forward and summing yields

$$C_{i,t}(j) = \Delta Y_{i,t} \beta \frac{1 - \beta^j}{1 - \beta} + \alpha \sum_{m=0}^{j-1} (j - m) \beta^m + \sum_{s=0}^{j-1} \frac{1 - \beta^{j-s}}{1 - \beta} u_{i,t+s}. \quad (9)$$

Define the weights

$$w_{j,s} := \frac{1 - \beta^{j-s}}{1 - \beta}, \quad s = 0, \dots, j - 1.$$

Let \mathcal{F}_t denote the information set generated by $\{\Delta Y_{i,s} : s \leq t\}$ (equivalently, by the history of $Y_{i,s}$ up to t). Conditionally on \mathcal{F}_t , the only random component of (9) is the linear combination of future shocks $\{u_{i,t+s}\}_{s=0}^{j-1}$, which are jointly normal and independent of \mathcal{F}_t . Therefore:

$$C_{i,t}(j) | \mathcal{F}_t \sim N(\mu_{i,t}(j), V(j)), \quad (10)$$

where

$$\mu_{i,t}(j) := \mathbb{E}[C_{i,t}(j) | \mathcal{F}_t] = \Delta Y_{i,t} \beta \frac{1 - \beta^j}{1 - \beta} + \alpha \sum_{m=0}^{j-1} (j - m) \beta^m, \quad (11)$$

$$V(j) := \text{Var}(C_{i,t}(j) | \mathcal{F}_t) = \sigma^2 \sum_{s=0}^{j-1} w_{j,s}^2 = \sigma^2 \sum_{s=0}^{j-1} \left(\frac{1 - \beta^{j-s}}{1 - \beta} \right)^2. \quad (12)$$

Thus, the cumulative change over j periods is conditionally Gaussian, with a mean that depends on the current difference $\Delta Y_{i,t}$ and parameters (α, β) , and a variance that aggregates the future innovation variance σ^2 through the weights $\{w_{j,s}\}$.

4.2.3 Unconditional distribution under stationarity

If $|\beta| < 1$, the differenced process (7) admits a unique covariance-stationary solution. In its stationary distribution,

$$\Delta Y_{i,t} \sim N\left(\mu_{\Delta}, \frac{\sigma^2}{1-\beta^2}\right), \quad \mu_{\Delta} := \frac{\alpha}{1-\beta}. \quad (13)$$

Moreover, $\Delta Y_{i,t}$ depends only on past shocks $\{u_{i,s}\}_{s \leq t-1}$, whereas $\{u_{i,t+s}\}_{s \geq 0}$ are future shocks, so $\Delta Y_{i,t}$ is independent of $\{u_{i,t+s}\}_{s=0}^{j-1}$.

Using (9), the unconditional mean is

$$\begin{aligned} \mathbb{E}[C_{i,t}(j)] &= \mathbb{E}[\mu_{i,t}(j)] \\ &= \mathbb{E}[\Delta Y_{i,t}] \beta \frac{1-\beta^j}{1-\beta} + \alpha \sum_{m=0}^{j-1} (j-m) \beta^m \\ &= \mu_{\Delta} \beta \frac{1-\beta^j}{1-\beta} + \alpha \sum_{m=0}^{j-1} (j-m) \beta^m. \end{aligned} \quad (14)$$

The unconditional variance follows from the independence of $\Delta Y_{i,t}$ and the future shocks:

$$\begin{aligned} \text{Var}(C_{i,t}(j)) &= \text{Var}\left(\Delta Y_{i,t} \beta \frac{1-\beta^j}{1-\beta}\right) + \text{Var}\left(\sum_{s=0}^{j-1} w_{j,s} u_{i,t+s}\right) \\ &= \beta^2 \left(\frac{1-\beta^j}{1-\beta}\right)^2 \text{Var}(\Delta Y_{i,t}) + \sigma^2 \sum_{s=0}^{j-1} w_{j,s}^2 \\ &= \beta^2 \left(\frac{1-\beta^j}{1-\beta}\right)^2 \frac{\sigma^2}{1-\beta^2} + \sigma^2 \sum_{s=0}^{j-1} \left(\frac{1-\beta^{j-s}}{1-\beta}\right)^2. \end{aligned} \quad (15)$$

Since $C_{i,t}(j)$ is an affine function of jointly normal variables $\Delta Y_{i,t}$ and $\{u_{i,t+s}\}_{s=0}^{j-1}$, it follows that

$$C_{i,t}(j) \sim N(\mathbb{E}[C_{i,t}(j)], \text{Var}(C_{i,t}(j))), \quad (16)$$

with mean and variance given by (14) and (15). A further exposition of this derivation is given in Appendix A.

4.3 Tracing M from Data

The age of maturity M is not directly observable. To trace it empirically, we estimate country-specific age fixed effects on first-differenced outcomes within two-year age bins. Concretely, for each outcome variable, we regress $\Delta y_{i,t}$ on its own lag and a full set of country-by-age-bin interaction dummies, omitting an intercept, over the age range 50 to 85. The estimated fixed effect for each country-age cell captures the average change in the outcome attributable to being in that age bin, net of individual-level persistence. M is then identified as the age bin at which the estimated fixed effect crosses zero from below and remains positive: the point past which, on average, the outcome deteriorates in every subsequent bin. Because mortality selection and cohort composition vary across countries, we allow the fixed effects to differ by country, and we require at least 20 observations per country-age cell to ensure reliable estimates. The resulting profiles, plotted with 95% confidence bands, allow us to assess both the level of M and its consistency across countries and outcome domains. In Appendix B.1.2, we show the plots of these analyses across our ageing variables.

5 Results

The regression results corresponding to Equation 3, shown in table 5.1, provide empirical support for the theoretical framework's allowance of temporary reversals in ageing variables while maintaining long-run Individ-

ual Monotonicity. The consistently negative and significant coefficients on lagged first differences, ranging from approximately -0.33 to -0.59 across all health and income measures and all specifications, indicate substantial mean reversion: deteriorations tend to be followed by partial recovery in subsequent periods. This pattern aligns with the theory's acknowledgement that fluctuations due to lifestyle changes or health shocks are permissible, and that ageing need not proceed monotonically at every time step.

The constants in the pooled specifications without individual fixed effects (Columns 2 and 4) confirm an underlying deterioration trend across all variables. For health outcomes, the positive and significant constants for the frailty index and functional disability indicate that these two ageing variables are, on average, increasing over time. For log real total gross income (Panel A), the negative and significant constant reflects declining income on average, which likewise constitutes deterioration in the relevant dimension. The pain index and mental health constants (Panels C and E) are positive and significant in the pooled specification but small and even only marginally significant once wave fixed effects are included, suggesting that part of the average deterioration in pain and mental health reflects common period effects rather than a structural ageing trend.

The share of individual fixed effects above zero ($\hat{\alpha}_i > 0$) varies considerably across outcomes. Functional disability shows the smallest share (0.181 and 0.162 in specifications 1 and 3 respectively), indicating that the vast majority of individuals have a non-positive individual-specific trend in disability changes, consistent with a strong tendency toward sustained deterioration. Pain and mental health show intermediate values, while income sits near 0.478, meaning roughly half of individuals have a positive individual-specific component in income changes.

Taken together, these results demonstrate that while Individual Inevitability ensures deterioration becomes certain over a sufficiently long horizon, the short-run path is not monotonic at every step, and there is meaningful heterogeneity in individual trajectories even as the long-run direction remains inevitable. The pattern of temporary reversibility within an overall declining trajectory appears to be a fundamental and consistent property of ageing variables across both health and economic domains.

Table 5.1: Dynamic Panel AR(1) Models: First-Difference Specifications

	(1)	(2)	(3)	(4)
Wave FE			✓	✓
Individual FE	✓		✓	
<i>Panel A: Log Real Total Gross Income ($\Delta \log Y$)</i>				
Lagged $\Delta \log Y$	-0.4782*** (0.0040)	-0.3886*** (0.0035)	-0.4780*** (0.0040)	-0.3742*** (0.0035)
Constant		-0.0584*** (0.0034)		-0.0159* (0.0085)
$\hat{\alpha}_i > 0$	0.478		0.478	
Observations	63,217	63,217	63,217	63,217
R ²	0.2465	0.1648	0.2466	0.1647
<i>Panel B: Frailty Index (ΔY)</i>				
Lagged ΔY	-0.5360*** (0.0033)	-0.3594*** (0.0028)	-0.5414*** (0.0032)	-0.3258*** (0.0028)
Constant		0.0247*** (0.0003)		0.0047*** (0.0012)
$\hat{\alpha}_i > 0$	0.638		0.304	
Observations	156,497	156,497	156,497	156,497
R ²	0.2301	0.0991	0.2535	0.0891
<i>Panel C: Pain Index (ΔY)</i>				
Lagged ΔY	-0.5893*** (0.0085)	-0.4648*** (0.0060)	-0.5893*** (0.0085)	-0.4648*** (0.0060)
Constant		0.0050*** (0.0011)		-0.0034 (0.0045)
$\hat{\alpha}_i > 0$	0.411		0.562	
Observations	22,308	22,308	22,308	22,308
R ²	0.3420	0.2147	0.3429	0.2154
<i>Panel D: Functional Disability (ΔY)</i>				
Lagged ΔY	-0.5643*** (0.0035)	-0.3937*** (0.0029)	-0.5678*** (0.0035)	-0.3630*** (0.0029)
Constant		0.0209*** (0.0004)		0.0117*** (0.0015)
$\hat{\alpha}_i > 0$	0.181		0.162	
Observations	152,703	152,703	152,703	152,703
R ²	0.2342	0.1077	0.2435	0.0917
<i>Panel E: Mental Health (ΔY)</i>				
Lagged ΔY	-0.5335*** (0.0030)	-0.4381*** (0.0024)	-0.5345*** (0.0030)	-0.4383*** (0.0024)
Constant		0.0088*** (0.0005)		-0.0040* (0.0022)
$\hat{\alpha}_i > 0$	0.436		0.406	
Observations	145,411	145,411	145,411	145,411
R ²	0.2845	0.1898	0.2875	0.1912

Standard errors in parentheses. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

All models estimated on pooled ELSA and SHARE data. The dependent variable is the first difference of each outcome (ΔY), except Panel A, where it is the first difference of the log ($\Delta \log Y$). The sole regressor is the lagged first difference. FE specifications use individual fixed effects. Wave FE refers to survey wave dummies. $\hat{\alpha}_i > 0$ reports the share of individual fixed effects greater than zero (FE specifications only).

5.1 The Distribution of Fixed Effects

Figure 11 presents the distribution of individual fixed effects (α_i) from the first-difference specifications across the five outcome variables. The distributions reveal striking heterogeneity in the degree and character of individual-specific variation across domains. The Frailty Index displays the most tightly concentrated distribution, with mass almost entirely within ± 0.05 and a sharp spike slightly above zero, indicating that individual-specific baseline deterioration rates are small but on average positive. Functional Disability similarly concentrates near zero within roughly ± 0.10 , but with visible secondary discrete mass points at multiples of approximately 0.25, reflecting the ordinal nature of functional limitation counts. Mental Health presents a broader distribution spanning roughly ± 1.0 with a moderate central spike and multiple secondary modes, reflecting the wider discrete scale of its symptom score. The Pain Index is the most structurally distinctive: it exhibits pronounced multimodality with several clearly separated peaks across approximately ± 0.5 , directly mirroring the discrete categories of the underlying measure and indicating that individual-specific pain trajectories cluster around a small number of distinct rates of change rather than being continuously distributed. Log Real Total Gross Income displays a markedly leptokurtic distribution: a sharp central spike at zero combined with heavy tails extending to approximately ± 5 log points, indicating that while most individuals have near-zero income trends after conditioning on the lagged dependent variable, a non-negligible share exhibit persistent positive or negative income drift. This leptokurtic character contrasts with the health variables, which have substantially narrower support, and reflects the episodic nature of large income changes in this elderly population.

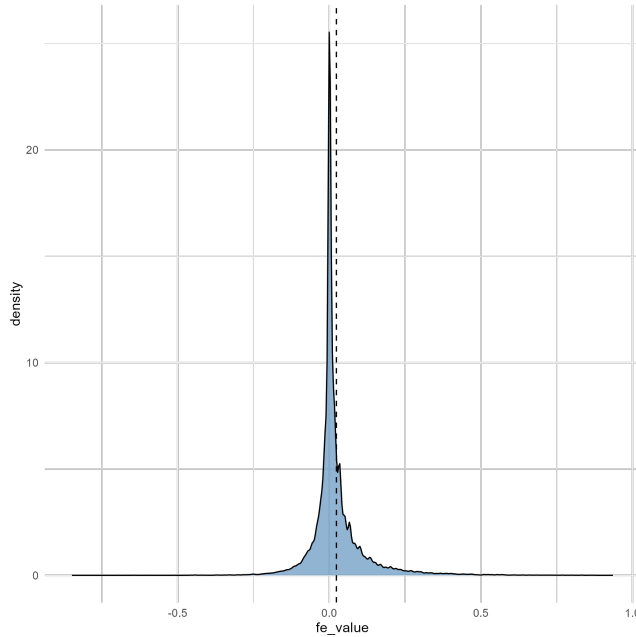


Figure 4: The Distribution of Fixed Effects: Frailty Index

5.2 Potential Malleability: Residual Autocorrelation and Observed Improvements

Figure 13 presents the distribution of individual-level first-order autocorrelation coefficients [ACF(1)] across two model specifications (FE Only and FE + Wave) for each of the five outcome variables. The distribution of individual-level residual autocorrelations reveals whether deterioration in ageing variables follows a common persistent dynamic or whether individuals differ fundamentally in how their trajectories evolve over time. The pronounced heterogeneity across individuals, obscured when residuals are aggregated, is itself evidence that a single summary of ageing dynamics is insufficient.

To complement this, Figure 14 directly quantifies observed improvements in each outcome. Beyond heterogeneity in dynamics, we can directly ask what fraction of individuals actually improve over a given horizon, which is the most direct empirical test of whether these variables satisfy Individual Inevitability. The results

confirm that for every variable studied, non-trivial shares of individuals experience genuine improvements, with the proportions varying systematically by variable, age, and horizon.

Lastly, our analyses in Figure 10 show how the unit root test at the individual level may reflect malleability: even though on average we observe deterioration, the individual path for all our ageing variables demonstrates substantial heterogeneity, not only with respect to the degree of deterioration, but also to potential for malleability and short-term, rare improvements. Individual-level stationarity tests provide a further lens on malleability: a stationary trajectory is one that reverts rather than drifts monotonically, which is incompatible with the kind of irreversible deterioration that Individual Inevitability would require. The distribution of test outcomes confirms that a substantial share of individuals exhibit stationary rather than unit-root behaviour. Also, those with a unit root may also exhibit a non-deteriorating trend (showing improvements in data over time).

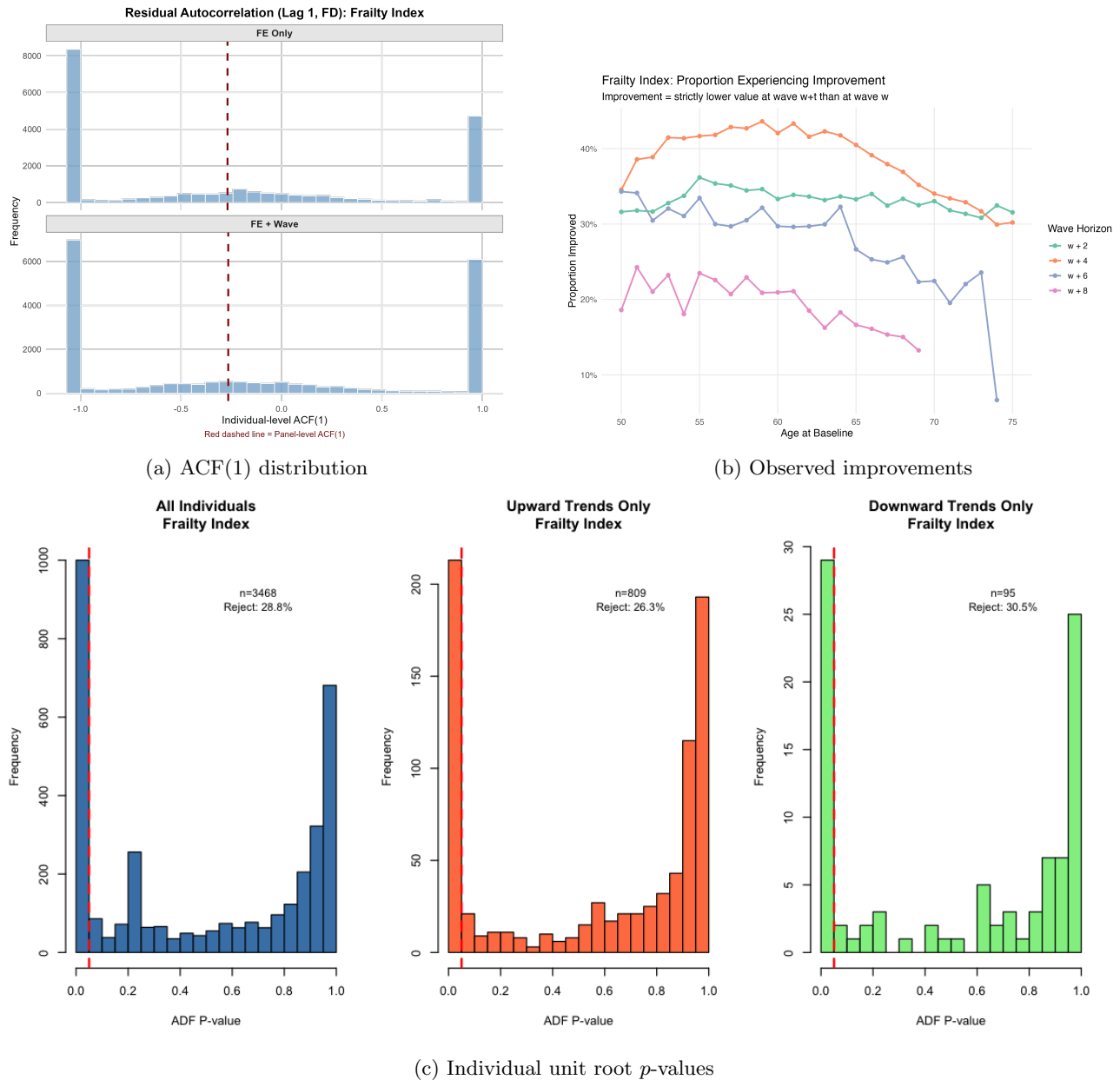


Figure 5: Malleability diagnostics for the frailty index. Panel (a) shows the distribution of individual-level first-order residual autocorrelations; panel (b) shows the fraction of individuals improving over horizons $h \in \{2, 4, 6, 8\}$ by age; panel (c) shows the distribution of individual ADF test p -values.

5.3 Testing the conditions for a given M

We now use the regression coefficients shown in column 2 of table 5.1 to estimate the expressions shown in Equation 16. In this section, as in the regressions, we take $M = 50$ as given. In the next section, we explore how these results change with M .

Each panel of Figure 6 reports the estimated mean and the associated 90% confidence interval for each variable across time horizons j . Visually, whether a variable can be characterised as exhibiting ageing behaviour depends on both the width of its confidence interval and the rate at which this width expands relative to the slope of the mean trajectory. Given the 90% confidence level, the interval corresponds to the 5th–95th percentiles of the distribution, which corresponds to a tolerance parameter (η) of 0.05.

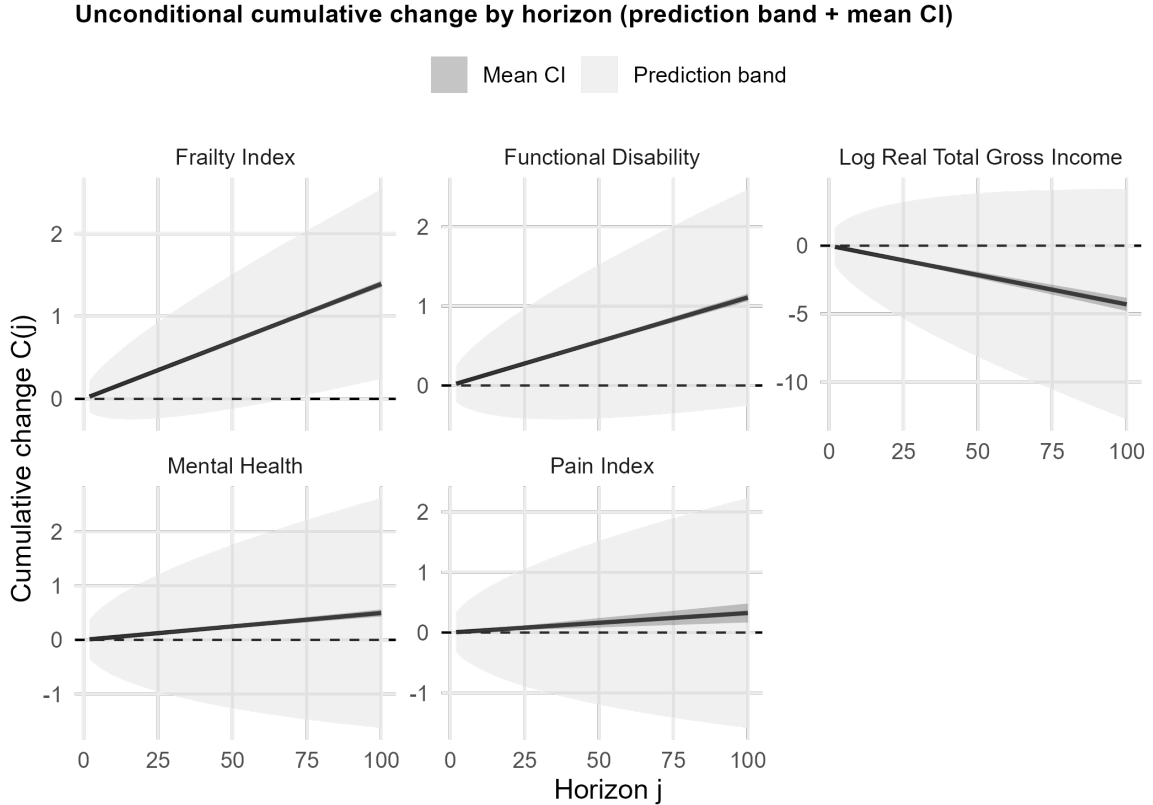


Figure 6: Confidence interval for the predicted cumulative change, by time horizon

Together, these can then be applied the condition shown in Equation 4. Table 5.2 reports the first time horizon j for 3 different values of η : 0.20, 0.10, and 0.05; the column corresponding to $\eta = 0.05$ aligns with Figure 6. The j shown corresponds to the minimal value of H in Equation 4 for different values of η . Except for the Frailty Index, the implied time horizons h at which the confidence interval excludes zero exceed a typical human lifespan. It is important to note that j indexes periods of two years rather than single years.

The results indicate that all variables satisfy the stated condition for some values of η . However, the implied horizons for log real total gross income, pain, index, and mental health are orders of magnitude larger than typical human lifespans, extending well beyond the range that could reasonably be extrapolated from the sample. By contrast, for variables such as frailty, the horizon is of a comparable order of magnitude: for example, at $\eta = 0.2$, the estimated horizon is $j = 30$, corresponding to 60 years; as M is set to 50 for this analysis, this corresponds to an age of 110. Although the timeline for functional disability exceeds the current plausible human lifespans, it remains closer to empirically meaningful scales.

These findings highlight several challenges inherent in testing for ageing effects in this setting. The variables under consideration are latent or limited dependent variables, and by the time the implied age thresholds are reached, they lie well outside the data's observable support. This raises concerns about the adequacy of linear

modelling in this context. While alternative specifications may offer improvements, incorporating lags and addressing potential unit-root behaviour substantially complicate estimation.

Table 5.2: First horizon j at which the unconditional prediction band excludes zero

	$\eta = 0.20$	$\eta = 0.10$	$\eta = 0.05$	
Log Real Total Gross Income	231	380	539	<i>Notes:</i> Entries report the smallest horizon j such that
Frailty Index	42	68	96	
Pain Index	2066	3403	N/A	
Functional Disability	92	150	213	
Mental Health	1098	1808	2566	

the prediction band for the unconditional cumulative change $C(j)$ excludes zero. For income variables, the lower bound is used (negative drift); for health outcomes, the upper bound is used.

5.4 Alternative choices of M

While the conditions require only existence of M , they do not imply a theoretical framework for finding M . One approach is to empirically consider the relationship between the implied inevitability timeline and current human lifespan. To investigate this, we find the value of M that minimises $M + 2j_{\eta=0.10}$, the age at which the individual passes the inevitability horizon, for each variable. M^* is capped at 85 due to data limitations. Table 5.3 shows those values of M and the corresponding values of H for each level of η . The conditions also require that both monotonicity and inevitability hold for all ages greater than M . We thus check that H exists for all subsequent ages. A checkmark in the corresponding column indicates that, at some later ages, an inevitability horizon could not be found and thus the condition is not satisfied. For Gross Income and Pain Index, it does not; these variables therefore do not meet the conditions to be considered ‘ageing variables’. Here, both Frailty Index and Functional Disability admit values of $M + 2j_{\eta=0.10}$ within observed human lifespans.

Table 5.3: Optimal age threshold M by variable (objective uses $\eta = 0.10$)

Variable	M^*	$M^* + 2j_{\eta=0.10}$	$j_{\eta=0.20}$	$j_{\eta=0.10}$	$j_{\eta=0.05}$	N/A later ($\eta = 0.20$)	N/A later ($\eta = 0.10$)	N/A later ($\eta = 0.05$)
Log Real Total Gross Income	65	483	127	209	296	Yes	Yes	Yes
Frailty Index	82	106	8	12	16	Yes	Yes	Yes
Pain Index	71	4451	1330	2190	3109	Yes	Yes	Yes
Functional Disability	82	130	15	24	34	Yes	Yes	Yes
Mental Health	78	420	104	171	241	Yes	Yes	Yes

Notes: M^* minimises $M + 2j_{\eta=0.10}$. ‘N/A later’ flags whether any subsequent age ($M + 1$ to 85) has $j = N/A$ at that η ; — when $M^* = 85$.

5.5 Implications for biological age

The conditions also shed light on the usefulness of “biological age”. This concept inherently assumes *monotonicity*. If a variable does not meet the ageing conditions, and for ages below M , biological age is not a meaningful concept. Non-monotonic or weakly increasing variables cannot be inverted to predict biological age. If, for example, a variable is non-monotonic up to age 75, the inversion will not be single-valued and therefore not produce a unitary value for biological age.

However, even if a given variable (or combination of variables) satisfies the ageing conditions, they may not convey sufficient information to be usefully translated to a population (average) measure. The conditions we develop relate to how an *individual’s state* evolves over time, while biological age measures relate an individual’s state to the average. For these transformations to be useful, the variation *across time* in individuals’ states must dominate the importance of their baseline. The role of each of these components in the individual levels and population variance of the Frailty Index is discussed in section 6.2.

6 Implications for measurement of ageing

In the following section, we use our model to understand the ageing process and how ‘ageing’ variables change over time.

6.1 Variance decompositions

First, we use the unit root specification to decompose the level of each variable over time. Previous work has discussed the evolution of ageing-related measures using a ‘slopes and intercepts’ framework (e.g. [Ritchie et al. \(2016\)](#)); in the unit root specification, $y_{i,M}$ is analogous to the intercept in this framework.

To decompose the variance, we first transform a model in levels to one in first differences

$$y_{i,t} = \gamma_i + \alpha_i t + \beta_1 y_{i,t-1} + \epsilon_{i,t}$$

noting that $y_{i,t-1} = \gamma_i + \alpha_i(t-1) + \beta_1 y_{i,t-2} + \epsilon_{i,t-1}$

$$\Delta y_{i,t} = \gamma_i - \gamma_i + \alpha_i t - \alpha_i(t-1) + \beta_1 y_{i,t-1} - \beta_1 y_{i,t-2} + \epsilon_{i,t} - \epsilon_{i,t-1}$$

$$\Delta y_{i,t} = \alpha_i + \beta \Delta y_{i,t-1} + \epsilon_{i,t}$$

The variance of $y_{i,t}$ in levels is made up of four components:

1. The starting point/initial value at $y_{i,0}$
2. The initial growth rate $\Delta y_{i,1}$
3. The deterministic process $\alpha_i + \beta \Delta y_{i,t-1}$
4. The error terms $\epsilon_{i,t}$

We can express $y_{i,t}$ as the sum of $y_{i,0}$ and all subsequent changes

$$y_{i,t} = y_{i,0} + \sum_{\tau=1}^t \Delta y_{i,\tau}$$

To decompose the $\sum_{\tau=1}^t \Delta y_{i,\tau}$ term we use the structure of the model to divide this into explained and unexplained parts.

$$\sum_{\tau=1}^t \Delta y_{i,\tau} = \underbrace{\Delta y_{i,1} \sum_{\tau=1}^t \beta^{\tau-1}}_{\text{contribution of initial change}} + \underbrace{\alpha_i \sum_{\tau=1}^t \sum_{k=0}^{\tau-1} \beta^k}_{\text{deterministic part}} + \underbrace{\sum_{\tau=1}^t \sum_{k=0}^{\tau-1} \beta^k \epsilon_{i,\tau-k}}_{\text{accumulated shocks}}$$

Going back to levels the total decomposition is thus

$$y_{i,t} = \underbrace{y_{i,0}}_{\text{(i) initial level}} + \underbrace{\Delta y_{i,1} \sum_{\tau=1}^t \beta^{\tau-1}}_{\text{(ii) contribution of initial change}} + \underbrace{\alpha_i \sum_{\tau=1}^{t-1} \sum_{k=0}^{\tau-1} \beta^k}_{\text{(iii) deterministic part}} + \underbrace{\sum_{\tau=1}^{t-1} \sum_{k=0}^{\tau-1} \beta^k \epsilon_{i,\tau-k}}_{\text{(iv) accumulated shocks}} \quad (17)$$

6.2 Decomposition - Frailty index

We illustrate this decomposition for the Frailty Index. First, we consider the contribution the average level of the variable over the course that individuals are observed. Figure 7 decomposes the average level into each of the components show in equation 17.⁷ For individuals of all starting ages, the majority of the level, even 14

⁷Note that the sample in this section is restricted to individuals present in waves 3, 4, 8, and 9. This is because the initial value and initial change must be observed, and to keep a consistent sample across the full period being examined. The regressions relating to this sample are shown in Table B.1. These results are broadly consistent with those for the full sample.

years on, comes from the initial value. Most of the increase in average value is accounted for by the deterministic portion of the process, while the impact of the initial change fades over time.

Figure 7: Portion of the average level attributed to each component - Frailty Index

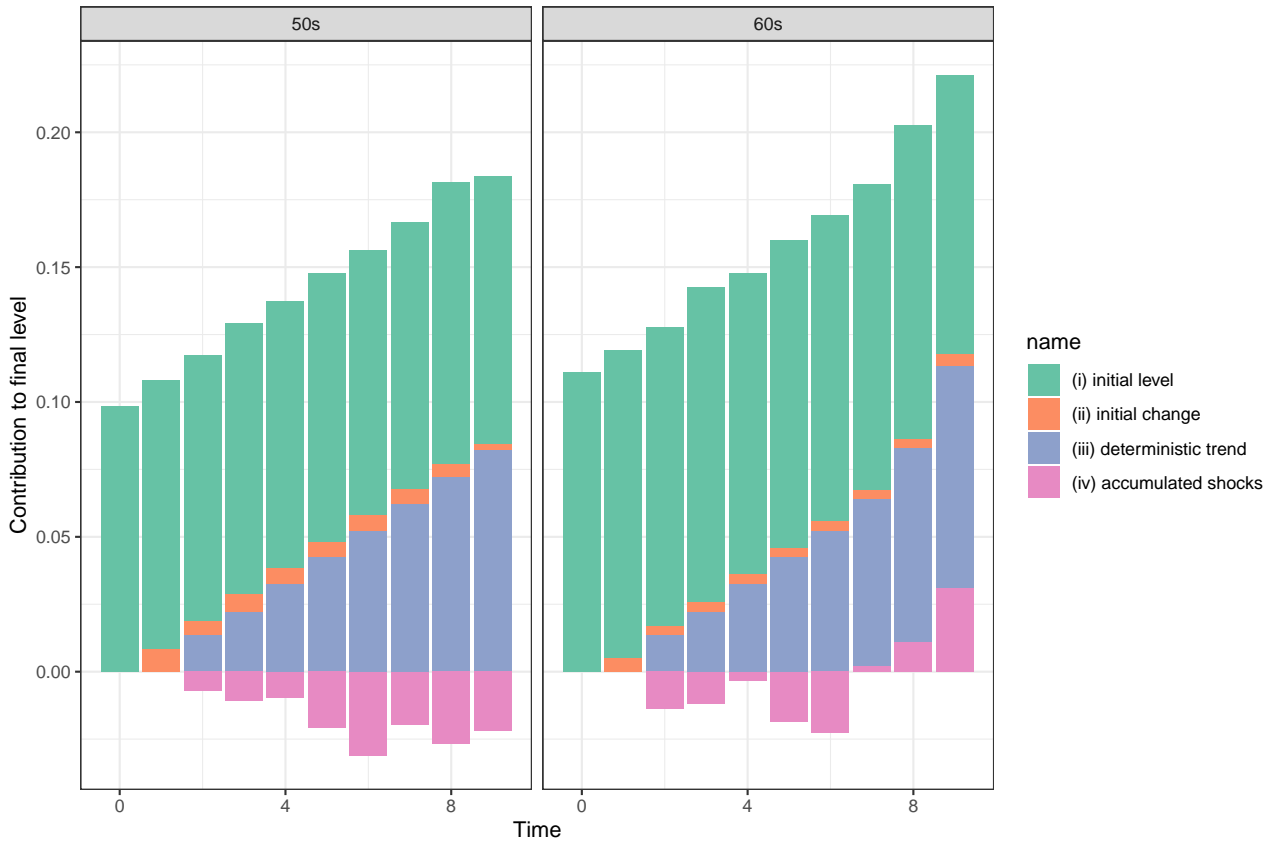
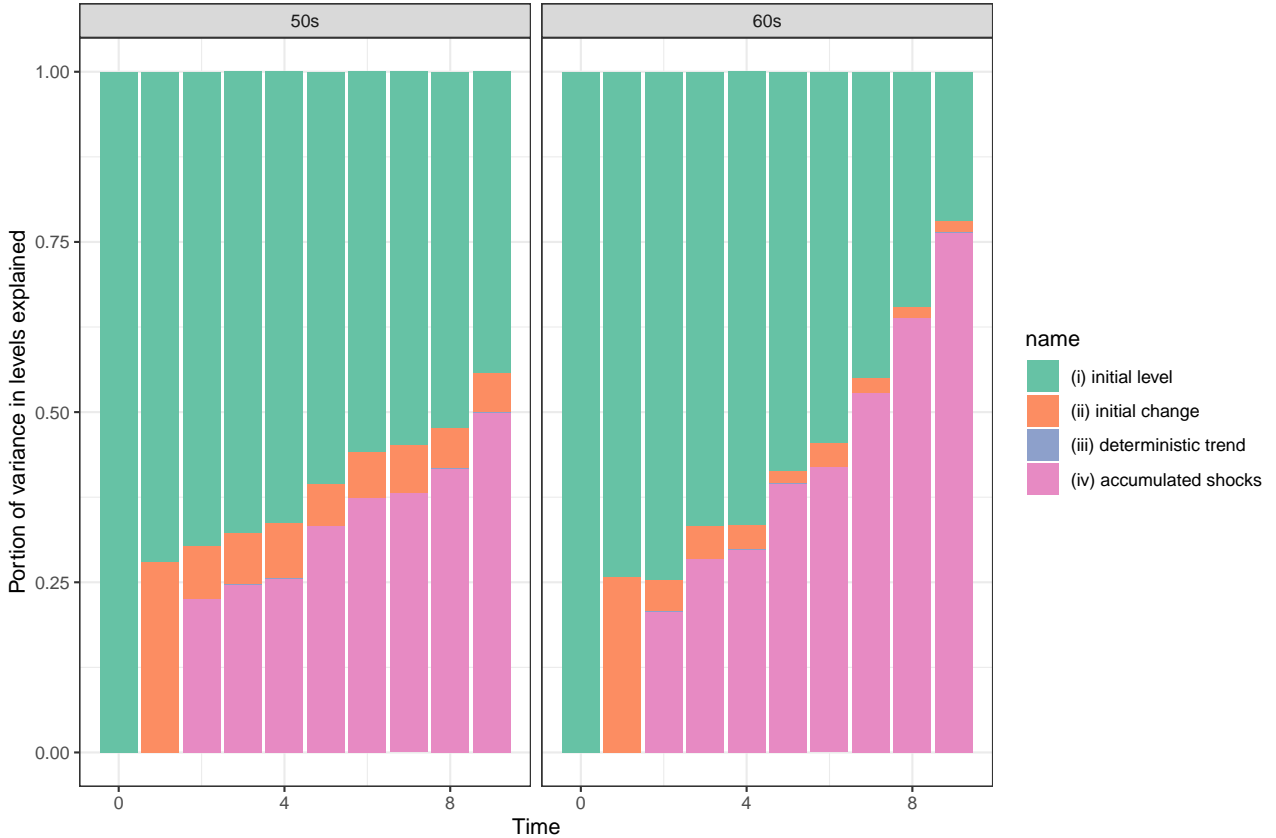


Figure 8 computes the share of the variance attributable to each component in each period, when α is common across all individuals. By construction when the deterministic trend is the same for all individuals, it does not contribute to the variance. In contrast to the average level, the accumulated shocks play a significant role in the variance between individuals. Nevertheless, even 7 periods (14 years) after the initial observation, half of the variance across individuals is attributable to the initial differences in levels. Because the specification uses a shared α , some of the variation attributed to accumulated shocks could instead be interpreted as differences in individual deterministic trend. Regardless of the interpretation of these differences as “random luck” or individual-specific trends, the level in the initial period remains the most important component, suggesting that health inequalities in older adults are not driven by differences in ageing, but rather differences in initial conditions.

This has important implications for the measurement of “biological age”. Even when individuals approach and pass the M^* calculated in section 5.4, the majority of their level is explained not by ageing, but by their own level at an earlier age. Thus, projecting these variables onto a shared ‘ageing’ scale is, at best, a noisy measure.

Figure 8: Decomposition of the variance in Frailty Index, with common slope



6.3 Non-Invertibility of Ageing Variables

A common approach in the biological age literature is to invert the relationship between a health measure and chronological age, using the estimated regression to read off an implied “biological age” for each individual. We assess the reliability of this inversion for each of our five outcome variables. For each variable, we estimate a simple panel regression of the outcome on chronological age under both random effects and fixed effects specifications, then invert the fitted relationship to compute a predicted age for each observation, adding measurement error equal to 1% of the variance of chronological age to simulate realistic noise in the health measure.

The results, shown in Appendix Figure 15, reveal that this inversion is unreliable across all variables. The scatter plots show that at any given chronological age, predicted ages span nearly the entire plausible range, with dispersion that does not narrow appreciably as individuals age. The spaghetti plots, which trace 50 randomly selected individuals over time, show that implied biological ages fluctuate erratically between waves, often shifting by decades within a single observation interval. This instability is not confined to income: it is present, if somewhat less extreme, for the frailty index, pain index, functional disability, and mental health symptoms as well.

These patterns follow directly from the individual heterogeneity documented earlier. If most of the cross-sectional and longitudinal variation in an outcome reflects differences in individual baselines established well before the age range under study, then the level of the outcome carries almost no information about where an individual stands relative to their own ageing trajectory. Inverting the population-level age gradient to construct a biological age implicitly assumes that all individuals share a common baseline, which our results reject. The unreliability of inversion is therefore not a measurement problem to be corrected but a structural consequence of the heterogeneity that motivates our axiomatic framework.

6.4 Cross-Country and Cross-Cohort Analyses

Appendix Figure 16 plots the age profiles of each outcome variable separately by country and by birth cohort, using weighted means within three-year age bins. The cross-country profiles show that the health variables (frailty, functional disability, and mental health symptoms) deteriorate broadly consistently with age across countries, though levels vary. Log income displays a non-monotonic profile, rising in early old age and declining sharply at higher ages, with considerable cross-country heterogeneity in both the level and the location of the turning point. The cross-country comparison illustrates that M and H may take different values across space, even at a given time.

The cohort profiles show that later-born cohorts tend to enter the age range in better condition for health variables, but trajectories converge at older ages, implying steeper deterioration over the observed window for more recent cohorts. This pattern is consistent with improvements in baseline health across cohorts while the underlying ageing gradient remains present. Given this pattern of improvement, a simple cross-sectional analysis would indicate that ageing happens more rapidly than it does within cohorts. In a cross-section, the difference between those at age 50 and age 70 includes not only the impact of ageing, but also the level differences across cohorts. Together, these also illustrate the difficulties with a single measure of biological age. People age differently across space and time, so relating all differences back to a single number, tied to a specific age, can be misleading.

7 Conclusion

In this paper, we establish an axiomatic framework for defining and measuring ageing based on two core conditions: monotonicity and inevitability. By distinguishing between individual, cohort, and population conceptions of ageing, we demonstrate that variables which appear to reflect ageing at the cross-sectional level may fail to satisfy individual-level ageing conditions. Our empirical analysis using ELSA data reveals four central findings with important implications for economic policy and research. First, the distinction between individual and population ageing matters critically for interpreting health and economic outcomes among older adults. Second, the majority of heterogeneity in later-life health and economic status reflects differences established in middle age rather than the ageing process itself, suggesting that policy attention should focus on reducing inequality earlier in life rather than treating age-related decline as the primary driver of disparate outcomes. Third, while ageing is inevitable over sufficiently long horizons, the high degree of age-independent heterogeneity and substantial mean reversion in health trajectories indicate considerable malleability in ageing processes over realistic time scales. Finally, we show that ageing variables are generally not reliably invertible, implying that measures of biological age extracted from health or economic indicators can be highly misleading even when those variables satisfy our proposed ageing conditions. These findings underscore the need for careful conceptual and empirical distinctions in ageing research and highlight the opportunities for interventions that address inequality and health trajectories before advanced age.

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Appendix

A Derivation of the Cumulative Change and Its Distribution

A.1 Closed-form solution for future differences

Starting from (7),

$$\Delta Y_{i,t+1} = \alpha + \beta \Delta Y_{i,t} + u_{i,t},$$

we obtain by forward iteration, for $j \geq 1$,

$$\begin{aligned} \Delta Y_{i,t+2} &= \alpha + \beta \Delta Y_{i,t+1} + u_{i,t+1} \\ &= \alpha + \beta(\alpha + \beta \Delta Y_{i,t} + u_{i,t}) + u_{i,t+1} \\ &= \alpha(1 + \beta) + \beta^2 \Delta Y_{i,t} + \beta u_{i,t} + u_{i,t+1}, \end{aligned} \tag{18}$$

and so on. By induction one can show that, for all $j \geq 1$ and $\beta \neq 1$,

$$\Delta Y_{i,t+j} = \beta^j \Delta Y_{i,t} + \alpha \sum_{m=0}^{j-1} \beta^m + \sum_{s=0}^{j-1} \beta^{j-1-s} u_{i,t+s}. \tag{19}$$

Using the geometric series $\sum_{m=0}^{j-1} \beta^m = (1 - \beta^j)/(1 - \beta)$ gives

$$\Delta Y_{i,t+j} = \beta^j \Delta Y_{i,t} + \alpha \frac{1 - \beta^j}{1 - \beta} + \sum_{s=0}^{j-1} \beta^{j-1-s} u_{i,t+s}. \tag{20}$$

A.2 Cumulative change as a function of initial state and shocks

The cumulative change over j periods is

$$C_{i,t}(j) = Y_{i,t+j} - Y_{i,t} = \sum_{h=1}^j \Delta Y_{i,t+h}. \tag{21}$$

Substituting (19) with j replaced by h ,

$$\begin{aligned} C_{i,t}(j) &= \sum_{h=1}^j \left[\beta^h \Delta Y_{i,t} + \alpha \frac{1 - \beta^h}{1 - \beta} + \sum_{s=0}^{h-1} \beta^{h-1-s} u_{i,t+s} \right] \\ &= \Delta Y_{i,t} \sum_{h=1}^j \beta^h + \alpha \sum_{h=1}^j \frac{1 - \beta^h}{1 - \beta} + \sum_{h=1}^j \sum_{s=0}^{h-1} \beta^{h-1-s} u_{i,t+s}. \end{aligned} \tag{22}$$

We simplify each term in turn.

Initial difference term. The first term in (22) is

$$\Delta Y_{i,t} \sum_{h=1}^j \beta^h = \Delta Y_{i,t} \beta \frac{1 - \beta^j}{1 - \beta}.$$

Constant α term. For the second term,

$$\begin{aligned}
\sum_{h=1}^j \frac{1-\beta^h}{1-\beta} &= \frac{1}{1-\beta} \sum_{h=1}^j (1-\beta^h) \\
&= \frac{1}{1-\beta} \left(j - \sum_{h=1}^j \beta^h \right) \\
&= \frac{1}{1-\beta} \left(j - \beta \frac{1-\beta^j}{1-\beta} \right), \tag{23}
\end{aligned}$$

which is one possible closed form. An equivalent and often more transparent representation is obtained by recognizing that

$$\sum_{h=1}^j \frac{1-\beta^h}{1-\beta} = \sum_{m=0}^{j-1} (j-m) \beta^m,$$

by rearranging the double sum over h and m . Hence the contribution of α can be written as

$$\alpha \sum_{m=0}^{j-1} (j-m) \beta^m,$$

which matches the main text.

Shock term. For the shock term, exchange the order of summation:

$$\begin{aligned}
\sum_{h=1}^j \sum_{s=0}^{h-1} \beta^{h-1-s} u_{i,t+s} &= \sum_{s=0}^{j-1} u_{i,t+s} \sum_{h=s+1}^j \beta^{h-1-s} \\
&= \sum_{s=0}^{j-1} u_{i,t+s} \sum_{m=0}^{j-1-s} \beta^m \\
&= \sum_{s=0}^{j-1} \frac{1-\beta^{j-s}}{1-\beta} u_{i,t+s}, \tag{24}
\end{aligned}$$

using again the geometric series. This yields the weights

$$w_{j,s} = \frac{1-\beta^{j-s}}{1-\beta}.$$

Collecting all pieces, we obtain

$$C_{i,t}(j) = \Delta Y_{i,t} \beta \frac{1-\beta^j}{1-\beta} + \alpha \sum_{m=0}^{j-1} (j-m) \beta^m + \sum_{s=0}^{j-1} \frac{1-\beta^{j-s}}{1-\beta} u_{i,t+s}, \tag{25}$$

which is equation (9) in the main text.

A.3 Conditional mean and variance

Conditionally on \mathcal{F}_t , both $\Delta Y_{i,t}$ and the deterministic α term are non-random, and the shocks $\{u_{i,t+s}\}_{s=0}^{j-1}$ are independent $N(0, \sigma^2)$. Since $C_{i,t}(j)$ is an affine function of normal variables, it is normal conditional on \mathcal{F}_t :

$$C_{i,t}(j) \mid \mathcal{F}_t \sim N(\mu_{i,t}(j), V(j)).$$

The mean follows immediately by dropping the zero-mean shock term:

$$\mu_{i,t}(j) = \Delta Y_{i,t} \beta \frac{1 - \beta^j}{1 - \beta} + \alpha \sum_{m=0}^{j-1} (j - m) \beta^m.$$

For the variance, use independence of the shocks and the fact that

$$\text{Var}\left(\sum_{s=0}^{j-1} w_{j,s} u_{i,t+s}\right) = \sum_{s=0}^{j-1} w_{j,s}^2 \text{Var}(u_{i,t+s}) = \sigma^2 \sum_{s=0}^{j-1} w_{j,s}^2,$$

which yields (12).

A.4 Unconditional mean and variance under stationarity

Under $|\beta| < 1$, the stationary distribution of $\Delta Y_{i,t}$ is given in (13).

$$C_{i,t}(j) = A_j \Delta Y_{i,t} + B_j + \sum_{s=0}^{j-1} w_{j,s} u_{i,t+s},$$

where

$$A_j := \beta \frac{1 - \beta^j}{1 - \beta}, \quad B_j := \alpha \sum_{m=0}^{j-1} (j - m) \beta^m.$$

Then

$$\begin{aligned} \mathbb{E}[C_{i,t}(j)] &= A_j \mathbb{E}[\Delta Y_{i,t}] + B_j = A_j \mu_\Delta + B_j, \\ \text{Var}(C_{i,t}(j)) &= A_j^2 \text{Var}(\Delta Y_{i,t}) + \text{Var}\left(\sum_{s=0}^{j-1} w_{j,s} u_{i,t+s}\right), \end{aligned}$$

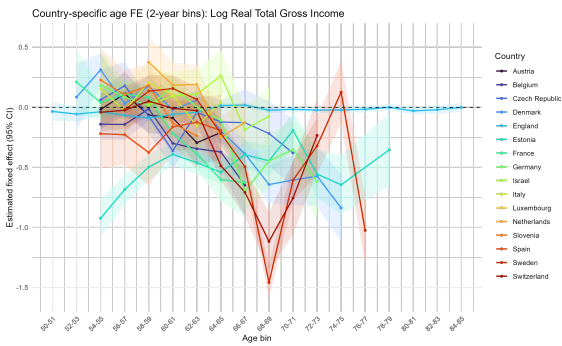
with no covariance term because $\Delta Y_{i,t}$ and the future shocks are independent. Substituting $\text{Var}(\Delta Y_{i,t}) = \sigma^2 / (1 - \beta^2)$ and $\text{Var}(\sum_{s=0}^{j-1} w_{j,s} u_{i,t+s}) = \sigma^2 \sum_{s=0}^{j-1} w_{j,s}^2$ gives the expressions reported in (14) and (15).

B Additional Graphs and Tables

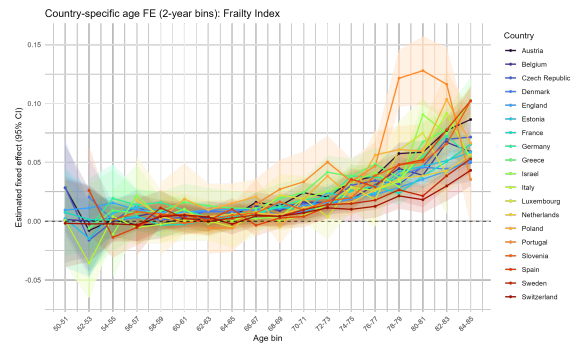
B.1 Additional Graphs

B.1.1 Unit Root Tests

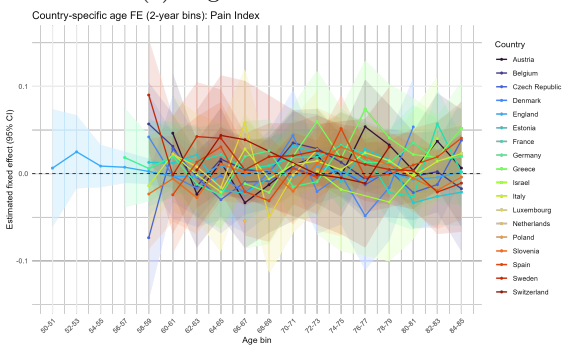
B.1.2 Tracing M



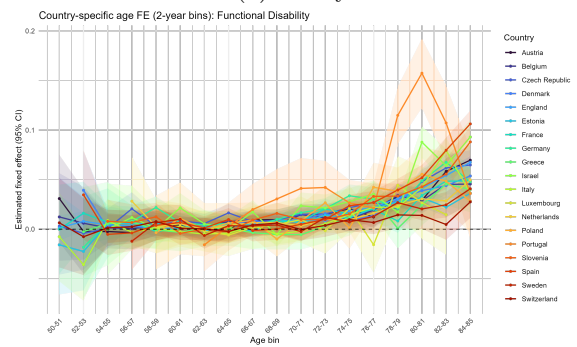
(a) Log Real Total Gross Income



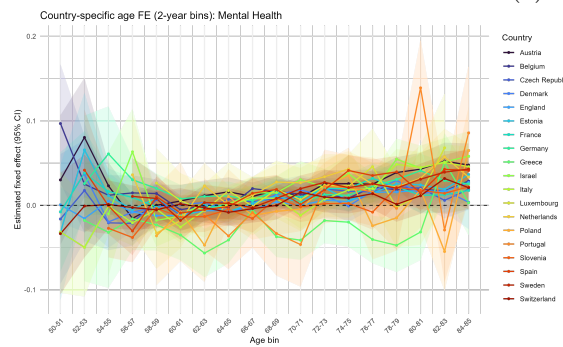
(b) Frailty Index



(c) Pain Index

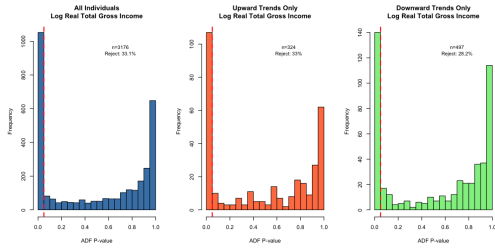


(d) Functional Disability

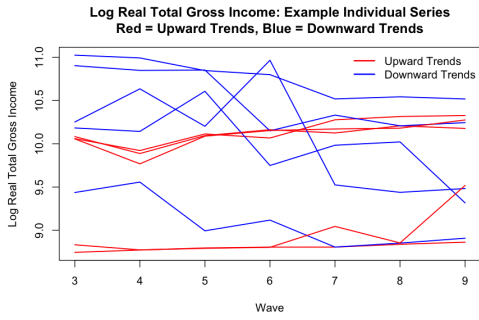


(e) Mental Health Symptoms

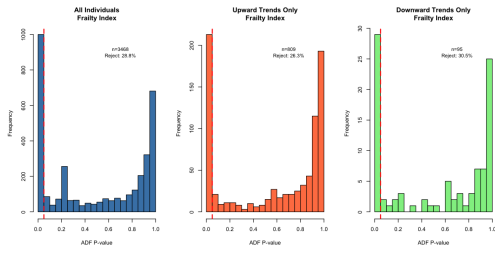
Figure 9: FE Densities



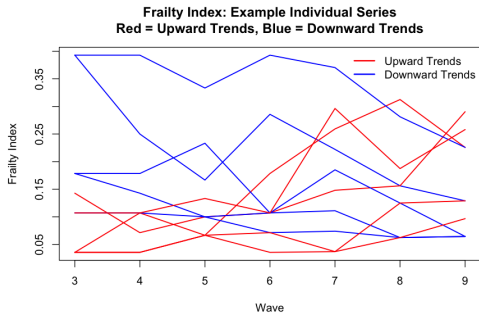
(a) Log Real Total Gross Income



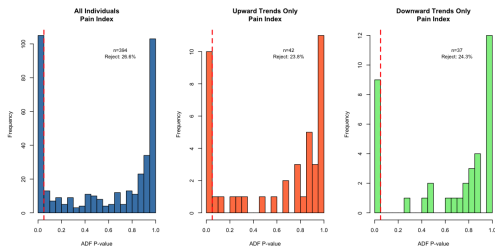
(b) Log Real Total Gross Income



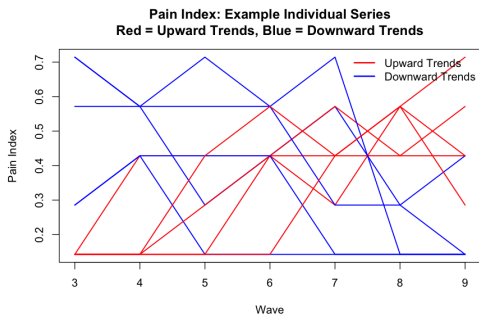
(c) Frailty Index



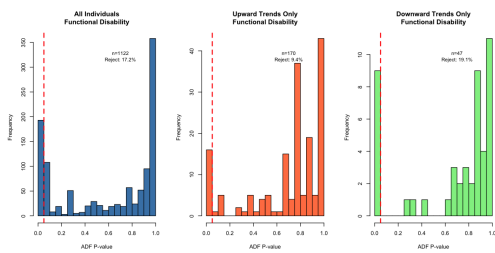
(d) Frailty Index



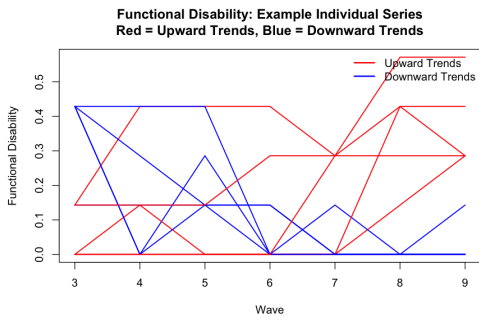
(e) Pain Index



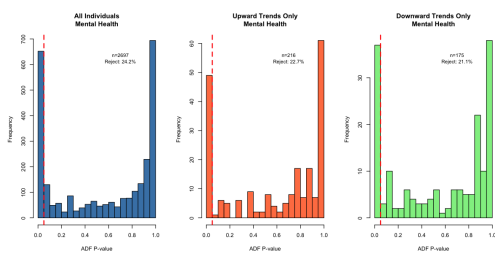
(f) Pain Index



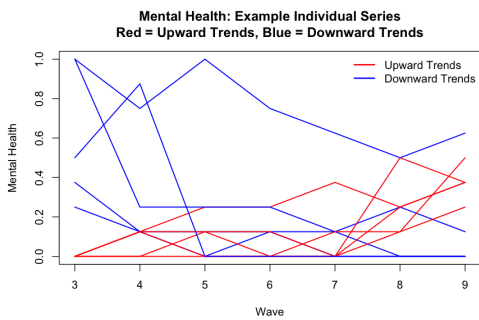
(g) Functional Disability



(h) Functional Disability



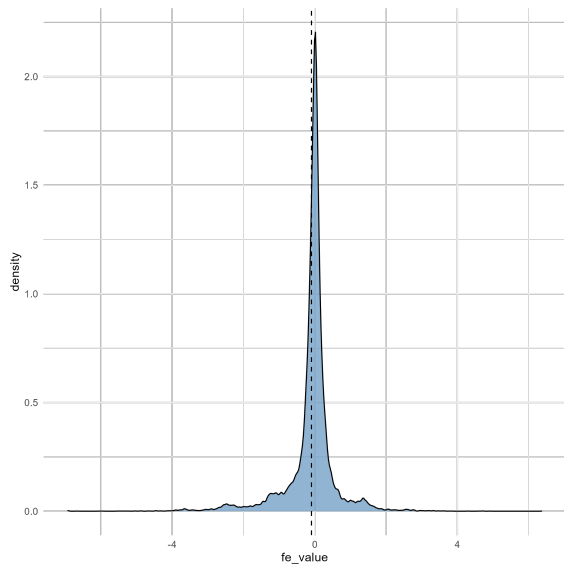
(i) Mental Health Symptoms



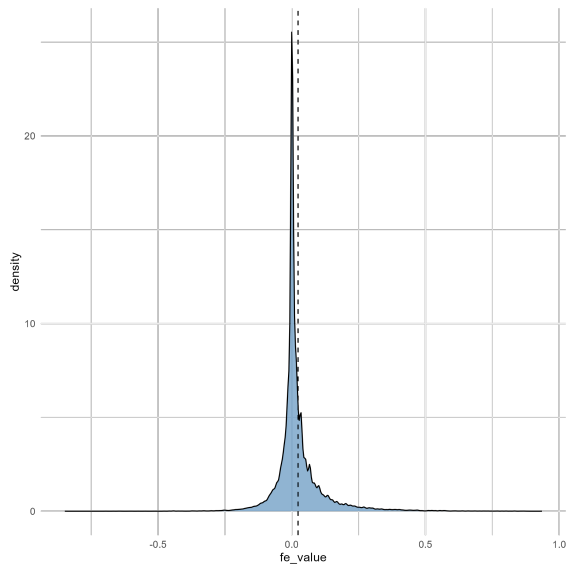
(j) Mental Health Symptoms

Figure 10: ADF P-Values Distribution and Examples of non-stationary individuals with Positive and Negative Slopes

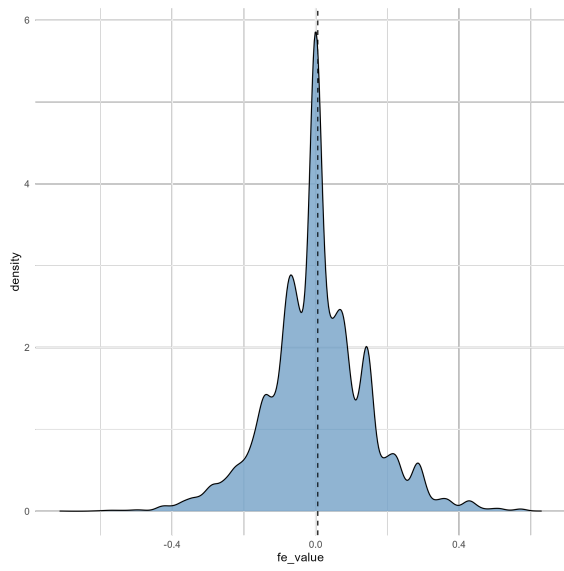
B.1.3 Fixed Effect Densities



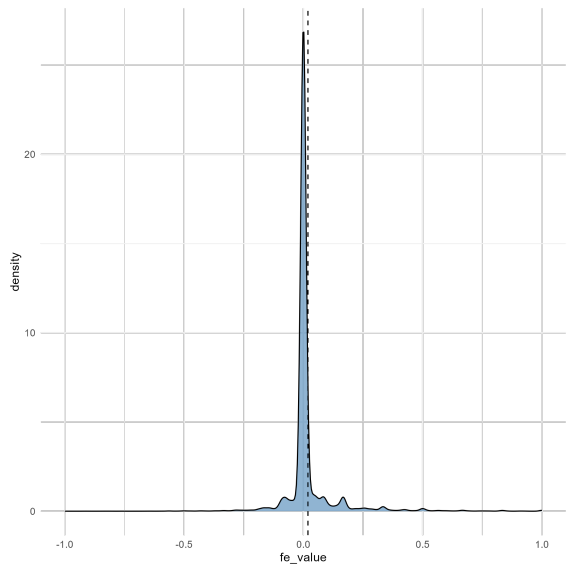
(a) Log Real Total Gross Income



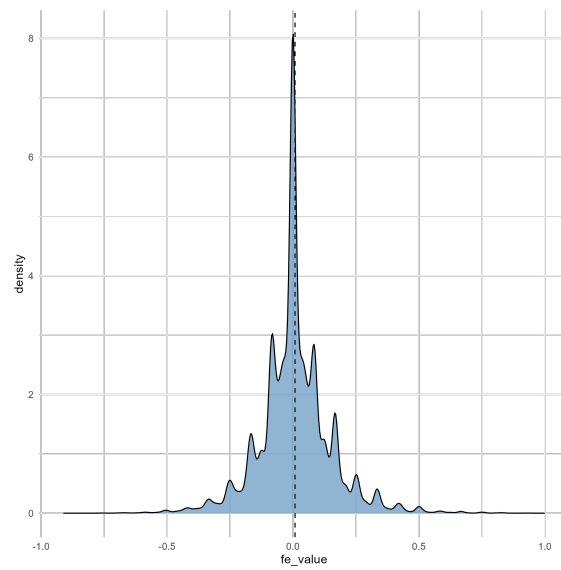
(b) Frailty Index



(c) Pain Index



(d) Functional Disability



(e) Mental Health Symptoms

Figure 11: FE Densities

B.1.4 Residual Distributions

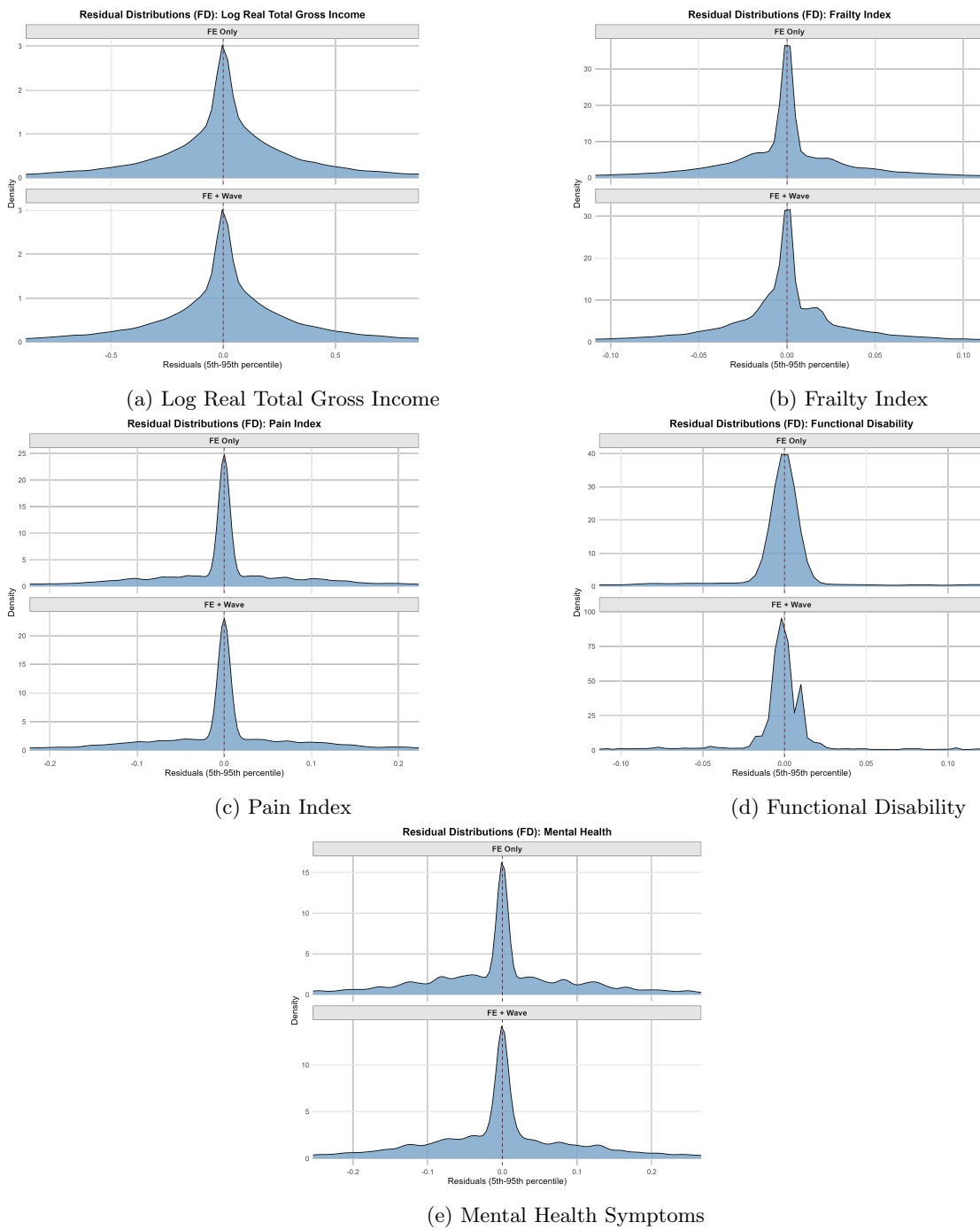


Figure 12: Residuals Distributions of Equation 3

B.1.5 Residual Autocorrelations

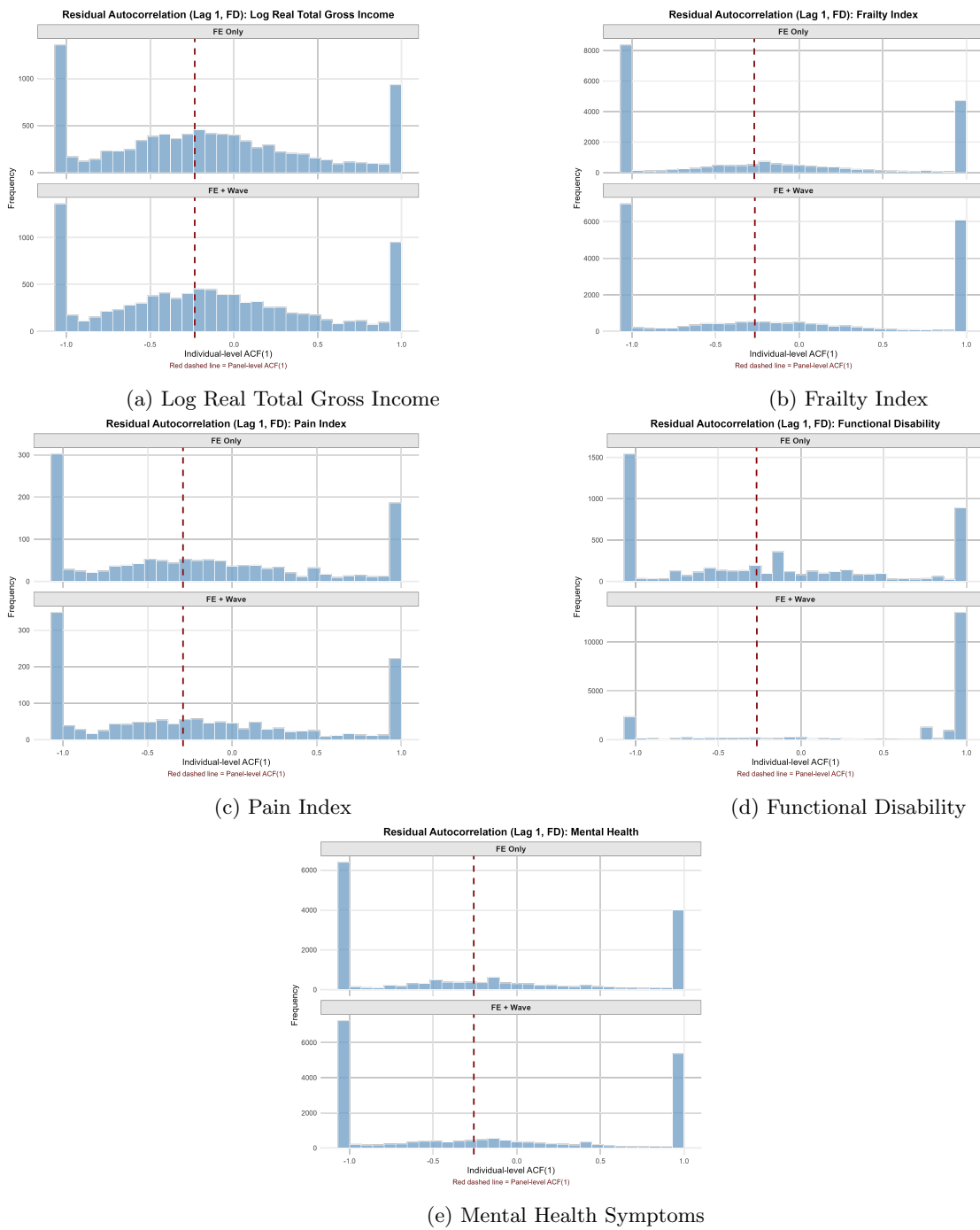
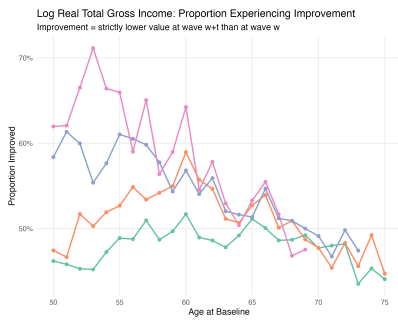
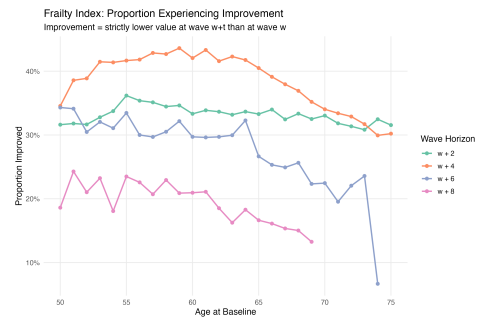


Figure 13: Individuals' Residuals Autocorrelation

B.1.6 Improvements



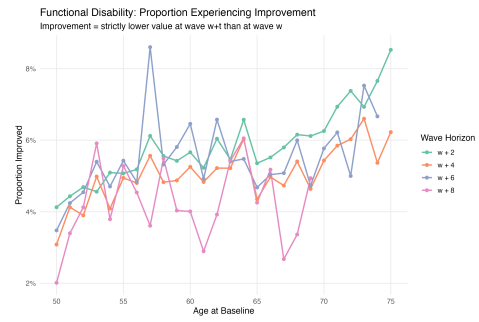
(a) Log Real Total Gross Income



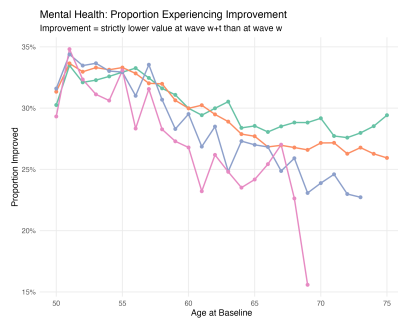
(b) Frailty Index



(c) Pain Index



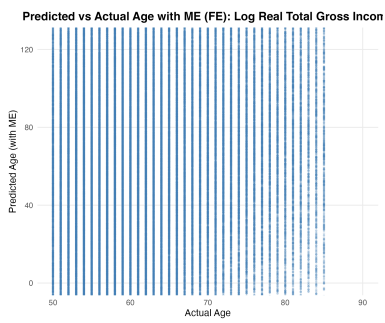
(d) Functional Disability



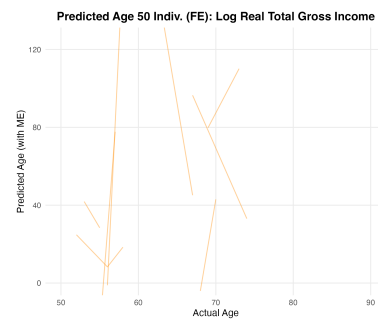
(e) Mental Health Symptoms

Figure 14: Improvements over Waves

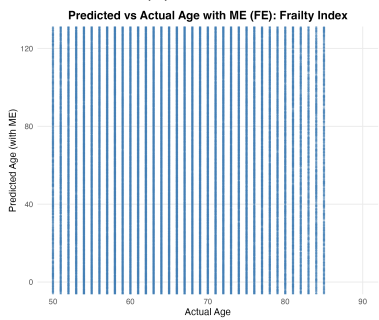
B.1.7 Non-Invertibility of Ageing Variables



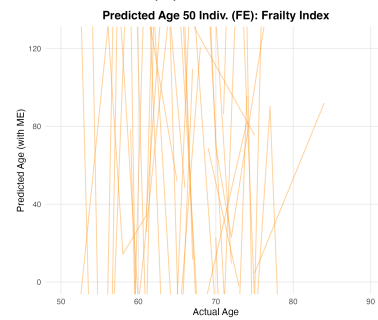
(a) Log Real Total Gross Income



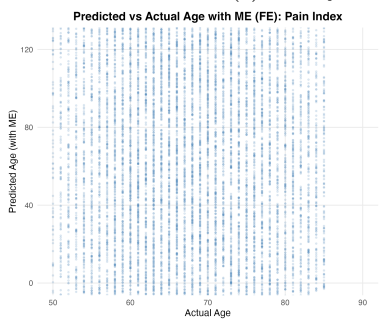
(b) Log Real Total Gross Income



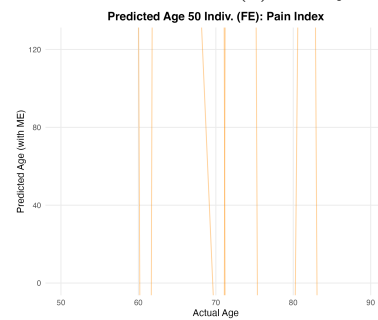
(c) Frailty Index



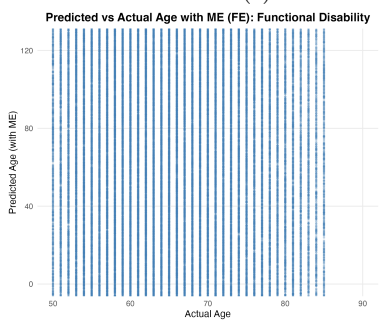
(d) Frailty Index



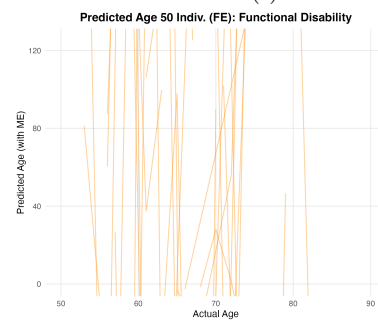
(e) Pain Index



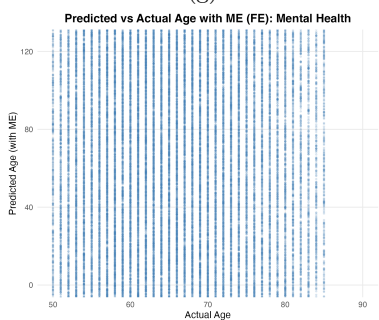
(f) Pain Index



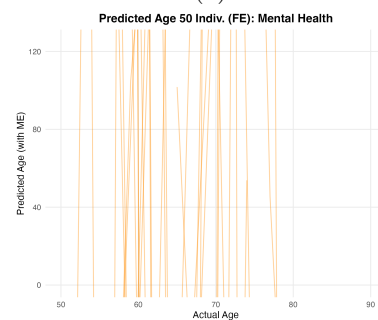
(g) Functional Disability



(h) Functional Disability



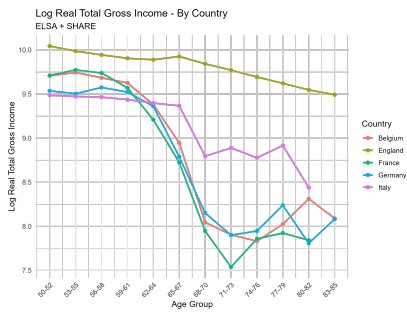
(i) Mental Health Symptoms



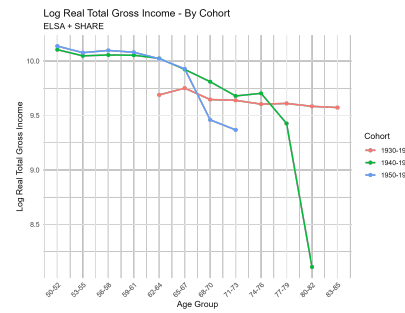
(j) Mental Health Symptoms

Figure 15: Scatter Plots of Observed Age and Predicted Age (On the left) and the Spaghetti Plots of Random Individuals (on the right)

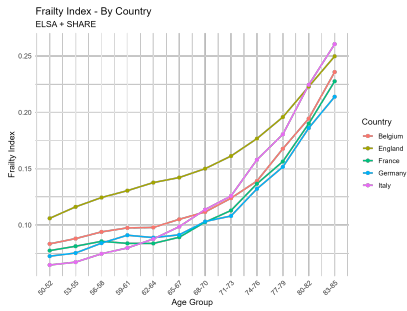
B.1.8 Cross-Country and Cross-Cohort Analysis



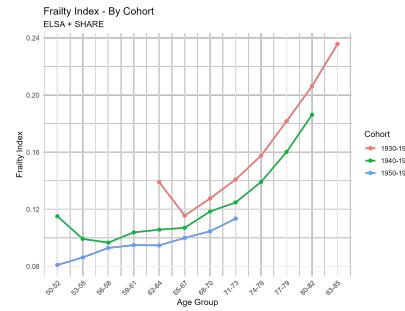
(a) Log Real Total Gross Income — By Country



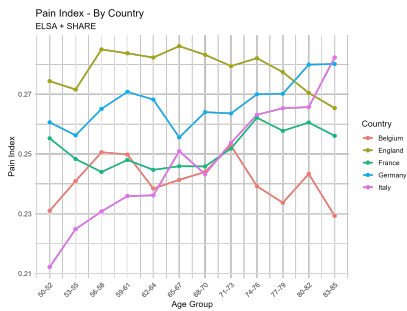
(b) Log Real Total Gross Income — By Cohort



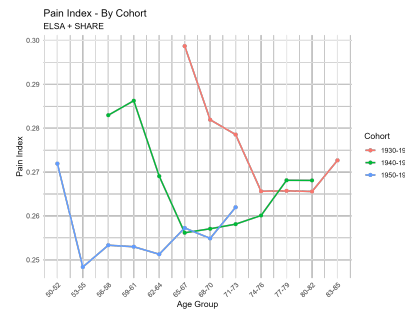
(c) Frailty Index — By Country



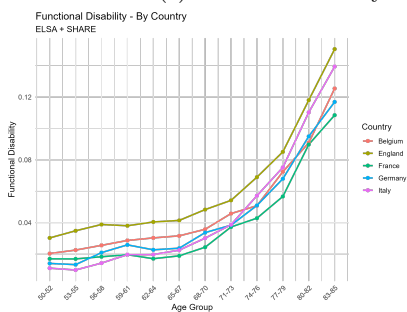
(d) Frailty Index — By Cohort



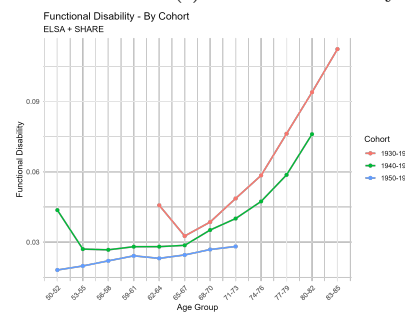
(e) Pain Index — By Country



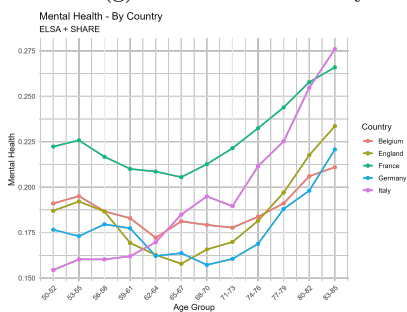
(f) Pain Index — By Cohort



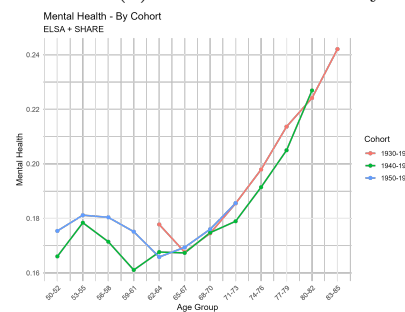
(g) Functional Disability — By Country



(h) Functional Disability — By Cohort



(i) Mental Health Symptoms — By Country



(j) Mental Health Symptoms — By Cohort

Figure 16: Age profiles of ageing variables by country (left) and birth cohort (right).

B.2 Additional Tables

B.2.1 Regression with Survivors (To be added also into a robustness Check)

Table B.1: Dynamic Panel AR(1) Models: First-Difference Specifications (Survivors Only) **UPDATED**

	(1)	(2)	(3)	(4)
Wave FE			✓	✓
Individual FE	✓		✓	
<i>Panel A: Log Real Total Gross Income ($\Delta \log Y$)</i>				
Lagged $\Delta \log Y$	-0.5082*** (0.0062)	-0.4709*** (0.0058)	-0.5082*** (0.0062)	-0.4710*** (0.0058)
Constant		-0.0474*** (0.0082)		-0.0271 (0.0241)
$\hat{\alpha}_i > 0$	0.422		0.381	
Observations	27,742	27,742	27,742	27,742
R ²	0.2287	0.1912	0.2291	0.1915
<i>Panel B: Frailty Index (ΔY)</i>				
Lagged ΔY	-0.4858*** (0.0052)	-0.3505*** (0.0048)	-0.4940*** (0.0051)	-0.3561*** (0.0048)
Constant		0.0169*** (0.0004)		0.0003 (0.0015)
$\hat{\alpha}_i > 0$	0.773		0.330	
Observations	46,521	46,521	46,521	46,521
R ²	0.1963	0.1019	0.2212	0.1226
<i>Panel C: Pain Index (ΔY)</i>				
Lagged ΔY	-0.5330*** (0.0128)	-0.4513*** (0.0103)	-0.5330*** (0.0129)	-0.4514*** (0.0103)
Constant		0.0042** (0.0020)		-0.0108* (0.0060)
$\hat{\alpha}_i > 0$	0.421		0.415	
Observations	7,498	7,498	7,498	7,498
R ²	0.2814	0.2032	0.2818	0.2046
<i>Panel D: Functional Disability (ΔY)</i>				
Lagged ΔY	-0.5048*** (0.0056)	-0.3667*** (0.0052)	-0.5098*** (0.0056)	-0.3714*** (0.0052)
Constant		0.0116*** (0.0005)		0.0009 (0.0018)
$\hat{\alpha}_i > 0$	0.194		0.192	
Observations	44,038	44,038	44,038	44,038
R ²	0.1949	0.1022	0.2030	0.1089
<i>Panel E: Mental Health (ΔY)</i>				
Lagged ΔY	-0.4863*** (0.0048)	-0.4352*** (0.0043)	-0.4870*** (0.0048)	-0.4366*** (0.0043)
Constant		0.0060*** (0.0009)		-0.0118*** (0.0032)
$\hat{\alpha}_i > 0$	0.459		0.399	
Observations	42,853	42,853	42,853	42,853
R ²	0.2400	0.1915	0.2432	0.1951

Standard errors in parentheses. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

All models estimated on pooled ELSA and SHARE data. Survivors-only sample includes individuals present in all waves 3, 4, 8, and 9. The dependent variable is first difference (ΔY), except Panel A where it is first difference of logs ($\Delta \log Y$). Wave FE uses survey-wave dummies. $\hat{\alpha}_i > 0$ is reported for FE specifications.

B.2.2 Regressions with Age/Cohort Fixed Effects (To be added also into a robustness Check)

B.2.3